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Abstract

This paper reports the results of tests on the weak-form market efficiency applied to the PSI-20 index prices of the Lisbon Stock Market from January 1993 to December 2006. As an emerging stock market, it is unlikely that it is fully information-efficient, but we show that the level of weak-form efficiency has increased in recent years. We use a serial correlation test, a runs test, an augmented Dickey-Fuller test and the multiple variance ratio test proposed by Lo and MacKinlay (1988) for the hypothesis that the stock market index follows a random walk. Non-trading or infrequent trading is not an issue because the PSI-20 only includes the 20 most traded shares. The tests are performed using daily, weekly and monthly returns for the whole period and for five sub-periods which reflect different trends in the market. We find mixed evidence, but on the whole, our results show that the Portuguese stock market index has been approaching a random walk behavior since year 2000, with a decrease in the serial dependence of returns. (JEL G14; G15)

Introduction

Efficient market theory and the random walk hypothesis have been major issues in financial literature, for the past thirty years. While a random walk does not imply that a market can not be exploited by insider traders, it does imply that excess returns are not obtainable through the use of information contained in the past movement of prices. The validity of the random walk hypothesis has important implications for financial theories and investment strategies, and so this issue is relevant for academicians, investors and regulatory authorities. Academicians seek to understand the behavior of stock prices, and standard risk-return models, such as the capital asset pricing model, depend of the hypotheses of normality or random walk behavior of prices. For investors, trading strategies have to be designed taking into account if the prices are characterized by random walks or by persistence in the short run, and mean reversion in the long run. Finally, if a stock market is not efficient, the pricing mechanism does not ensure the efficient allocation of capital within an economy, with negative effects for the overall economy. Evidence of inefficiency may lead regulatory authorities to take the necessary steps and reforms to correct it.

Since the seminal work of Fama (1970), several studies show that stock price returns do not follow a random walk and are not normally distributed, including Fama and French (1988) and Lo and MacKinlay (1988), among many others. The globalization markets spawned interest on the study of this issue, with many studies both on individual markets and regional markets, such as Latin America (Urrutia 1995, Grieb and Reyes 1999), Africa (Smith at al. 2002, Magnusson and Wydick 2002), Asia (Huang 1995, Groenewold and Ariff 1998), Middle East (Abraham et al. 2002) and Europe (Worthington and Higgs 2004), reporting unconformity with random walk behavior. The list is too extensive for a comprehensive survey, which is beyond the purpose of this study.

Previous studies of weak-form efficiency of the Portuguese market include Gama (1998), Dias et al. (2002), Smith and Ryoo (2003) and Worthington and Higgs (2004). Both Gama (1998) and Smith and Ryoo (2003) use a variance ratio test and conclude that the Portuguese market was not weak-form efficient until 1998. To our knowledge, the most complete study on Portugal until now is Dias et al. (2002) who study daily data of the PSI-20 index from January 1993 to September 2001 and find favorable evidence for a random walk by an augmented Dickey-Fuller test, but find stronger evidence against this hypothesis, using serial correlation and variance ratio tests. Worthington and Higgs (2004) use more recent data, from August 1995 to May 2003, and mostly find evidence that does not allow the rejection of a random walk, using serial correlation, augmented Dickey-Fuller and variance-ratio tests.

The main contribution of this paper is to add to international evidence on the random walk theory of stock market prices, by testing the Portuguese benchmark index (PSI-20), for the null hypothesis of a random walk. It adds on previous studies for the Portuguese stock market, by demonstrating that the evolution in recent years, until 2006, has been in the direction of increased weak-form efficiency.

Methodology

Serial correlation of returns

An intuitive test of the random walk for an individual time series is to check for serial correlation. If the PSI-20 index returns exhibit a random walk, the returns are uncorrelated at all leads and lags. We perform least square regressions of daily, weekly and monthly returns on lags one to ten of the return series. To test the joint hypothesis that all serial coefficients $\rho(t)$ are simultaneously equal to zero, we apply the Box-Pierce Q statistic:

$$Q_{BP} = n\sum_{t=1}^{m} \hat{\rho}(t) \tag{1}$$

where Q_{BP} is asymptotically distributed as a chi-square with m degrees of freedom, n is the number of observations, and m is the maximum lag considered (in this study, m equals ten). We also use a Ljung-Box test, which provides a better fit to the chi-square distribution, for small samples:

$$Q_{LB} = n(n+2)\sum_{t=1}^{m} \frac{\hat{\rho}^2(t)}{n-t} \tag{2}$$

Runs test

To test for serial independence in the returns we also employ a runs test, which determines whether successive price changes are independent of each other, as should happen under the null hypothesis of a random walk. By observing the number of runs, that is, the successive price changes (or returns) with the same sign, in a sequence of successive price changes (or returns), we can test that null hypothesis. We consider two approaches: in the first, we define as a positive return (+) any return greater than zero, and a negative return (-) if it is below zero; in the second approach, we classify each return according to its position with respect to the mean return of the period under analysis. In this last approach, we have a positive (+) each time the return is above the mean return and a negative (-) if it is below the mean return. This second approach has the advantage of allowing for and correcting the effect of an eventual time drift in the series of returns. Note that this is a non-parametric test, which does not require the returns to be normally distributed. The runs test is based on the premise that if price changes (returns) are random, the actual number of runs (R) should be close to the expected number of runs (μ_R).

Let n_+ and n_- be the number of positive returns (+) and negative returns (-) in a sample with n observations, where $n = n_+ + n_-$. For large sample sizes, the test statistic is approximately normally distributed:

$$Z = \frac{R - \mu_R}{\sigma_R} \approx N(0,1) \qquad (3)$$

where $\mu_R = \frac{2n_+n_-}{n} + 1$ and $\sigma_R = \sqrt{\frac{2n_+n_-(2n_+n_- - n)}{n^2(n-1)}}$.

Unit Root Tests

Our third test is the augmented Dickey-Fuller (ADF) test which is used to test the existence of a unit root in the series of price changes in the stock index series, by estimating the following equation through OLS:

$$\Delta P_t = \alpha_0 + \alpha_1 t + \rho_0 P_{t-1} + \sum_{i=1}^{q} \rho_i \Delta P_{it-i} + \varepsilon_{it} \qquad (4)$$

where P_t is the price at time t, and $\Delta P_t = P_t - P_{t-1}$, ρ_i are coefficients to be estimated, q is the number of lagged terms, t is the trend term, α_i is the estimated coefficient for the trend, α_0 is the constant, and ε is white noise. The null hypothesis of a random walk is $H_0 : \rho_0 = 0$ and its alternative hypothesis is $H_1 : \rho_0 \neq 0$. Failing to reject H_0 implies that we do not reject that the time series has the properties of a random walk. We use the critical values of MacKinnon (1994) in order to determine the significance of the *t*-statistic associated with ρ_0.

Variance Ratio Tests

An important property of the random walk is explored by our final test, the variance ratio test. If P_t is a random walk, the ratio of the variance of the q^{th} difference scaled by q to the variance of the first difference tends to equal one, that is, the variance of the q-differences increases linearly in the observation interval,

$$VR(q) = \frac{\sigma^2(q)}{\sigma^2(1)} \tag{5}$$

where $\sigma^2(q)$ is $1/q$ the variance of the q-differences and $\sigma^2(1)$ is the variance of the first differences. Under the null hypothesis $VR(q)$ must approach unity. The following formulas are taken from Lo and MacKinlay [1988], who propose this specification test, for a sample size of $nq+1$ observations ($P_0, P_1, ..., P_{nq}$):

$$\sigma^2(q) = \frac{1}{m}\sum_{t=q}^{nq}\left(P_t - P_{t-q} - q\hat{\mu}\right)^2 \tag{6}$$

where $m = q(nq-q+1)\left(1-\frac{q}{nq}\right)$ and $\hat{\mu}$ is the sample mean of $(P_t - P_{t-1})$: $\hat{\mu} = \frac{1}{nq}\left(P_{nq} - P_0\right)$ and

$$\sigma^2(1) = \frac{1}{(nq-1)}\sum_{t=1}^{nq}\left(P_t - P_{t-1} - \hat{\mu}\right)^2 \tag{7}$$

Lo and MacKinlay (1988) generate the asymptotic distribution of the estimated variance ratios and propose two test statistics, $Z(q)$ and $Z^*(q)$, under the null hypothesis of homoskedastic increments random walk and heteroskedastic increments random walk respectively. If the null hypothesis is true, the associated test statistic has an asymptotic standard normal distribution. Assuming homoskedastic increments, we have

$$Z(q) = \frac{VR(q) - 1}{\phi_o(q)} \approx N(0,1) \tag{8}$$

where $\phi_o(q) = \left[\frac{2(2q-1)(q-1)}{3q(nq)} \right]^{1/2}$. Assuming heteroskedastic increments, the test statistic is

$$Z^*(q) = \frac{VR(q) - 1}{\phi_e(q)} \approx N(0,1) \tag{9}$$

where $\phi_e(q) = \left[4 \sum_{t=1}^{q-1} \left(1 - \frac{t}{q} \right) \hat{\delta}_t \right]^{1/2}$ and $\hat{\delta}_t = \frac{\sum_{j=t+1}^{nq} (P_j - P_{j-1} - \hat{\mu})^2 (P_{j-t} - P_{j-t-1} - \hat{\mu})^2}{\left[\sum_{j=1}^{nq} (P_j - P_{j-1} - \hat{\mu})^2 \right]^2}$.

which is robust under heteroskedasticity, hence can be used for a longer time series analysis. The procedure proposed by Lo and MacKinlay (1988) is devised to test individual variance ratio tests for a specific q-difference, but under the random walk hypothesis, we must have $VR(q) = 1$ for all q. A multiple variance ratio test is proposed by Chow and Denning (1993). Consider a set of m variance ratio tests $\{M_r(q_i) | i = 1,2,...,m\}$ where $M_r(q) = VR(q) - 1$, associated with the set of aggregation intervals $\{q_i | i = 1,2,...,m\}$. Under the random walk hypothesis, there are multiple sub-hypotheses:

$H_{0i} : M_r(q_i) = 0$ for $i = 1,2,...,m$

$H_{1i} : M_r(q_i) \neq 0$ for any $i = 1,2,...,m$

The rejection of any or more H_{0i} rejects the random walk null hypothesis. In order to facilitate comparison of this study with previous research (Lo and MacKinlay, 1988 and Campbell *et al.* 1997) on other markets, the q is selected as 2, 4, 8, and 16. For a set of test statistics $\{Z(q_i) | i = 1,2,...,m\}$, the random walk hypothesis is rejected if any one of the $VR(q_i)$ is significantly different than one, so only the maximum absolute value in the set of test statistics is considered. The Chow and Denning (1993) multiple variance ratio test is based on the result:

$$PR\left\{\max\left(\left|Z(q_1)\right|,\ldots,\left|Z(q_m)\right|\right)\leq SMM(\alpha;m;T)\right\}\geq 1-\alpha \quad (10)$$

in which $SMM(\alpha;m;T)$ is the upper α point of the Studentized Maximum Modulus (*SMM*) distribution with parameters *m* and *T* (sample size) degrees of freedom. Asymptotically,

$$\lim_{T\to\infty} SMM(\alpha;m;\infty)=Z_{\alpha^*/2} \quad (11)$$

where $Z_{\alpha^*/2}$ is standard normal with $\alpha^* = 1-(1-\alpha)^{1/m}$. Chow and Denning (1993) control the size of the multiple variance ratio test by comparing the calculated values of the standardized test statistics, either $Z(q)$ or $Z^*(q)$ with the *SMM* critical values. If the maximum absolute value of , say, $Z(q)$ is greater than the critical value at a predetermined significance level then the random walk hypothesis is rejected.

The Data

Our data are daily closing values of the PSI-20 index, which is the Portuguese benchmark index, a cap weighted index reflecting the evolution of the prices of the 20 largest and most liquid shares selected from the universe of companies listed on the Portuguese Main Market. The PSI-20 also serves the purpose of acting as the underlying for futures and options contracts and other index linked products. The source of all data is *Reuters*, and it includes observations from 1 January 1993 to 31 December 2006, during which the index has shown significant fluctuations, as shown in Figure 1.

We apply the tests to the whole sample, but also separately to five periods which are defined by different trends in the market index. In period 1 (from 1-Jan-1993 to 31-Dec-1996) the index showed a trend of slow growth, which accelerated in the period 2 (from 2-Jan-1997 to 22-Apr-1998) reaching a peak in this last day. In period 3 (from 23-Apr-1998 to 10-Mar-2000) the index first declined sharply, and then grew very strongly reaching an all-time peak on 10-Mar-2000. Period 4 (from 13-Mar-2000 to 30-Sep-2002) was a depressive period for the market, with a steady trend of descent. In the final period (from 01-Oct-2002 to 29-Dec-

2006) the market again recovered a steady growth trend. It is important to clarify that this sub-periods are not defined in terms of any institutional change, and do not reflect any statistical criteria; it is a naïve criterion reflecting only visual trend changes of the market. The testing of periods has also the advantage of allowing for structural changes, so that the market may follow a random walk in some of the periods while in other periods that hypothesis may be rejected. A similar approach of arbitrarily-chosen periods is taken by Wheeler et al. (2002) in their analysis of the Warsaw Stock Exchange. Roughly, each of the periods has duration from one and a half years to four years. We are particularly interested in period 5, from March 2003 to December 2006, both because it has not been covered by previous studies, and because it is "now".

FIGURE 1
Lisbon Stock Market PSI 20 Index – Closing Prices

Non-trading is not a problem for the statistical tests since all the companies included in the index are only very rarely not traded on any given day, and the index is bound to fluctuate on every trading day. We use the daily closing prices to compute also weekly and monthly data. The weekly price series is constructed with the closing price on Wednesdays, to minimize day-of-the-week effects. If the Wednesday observation is not available, due to market closing, we use the Tuesday observation, and if that is also not available, we use

Thursday. For the monthly price series, we use the observations of day 15 of each month. In case of a missing observation on day 15, we use day 14. If day 14 is missing, we use day 16. If day 16 is missing we use day 13, and so on. From the sample of 3490 daily observations, we generate 730 weekly observations and 168 monthly observations. The returns are computed as the logarithmic difference between two consecutive prices in a series. Table 1 shows the descriptive statistics for the returns of the PSI-20 stock index.

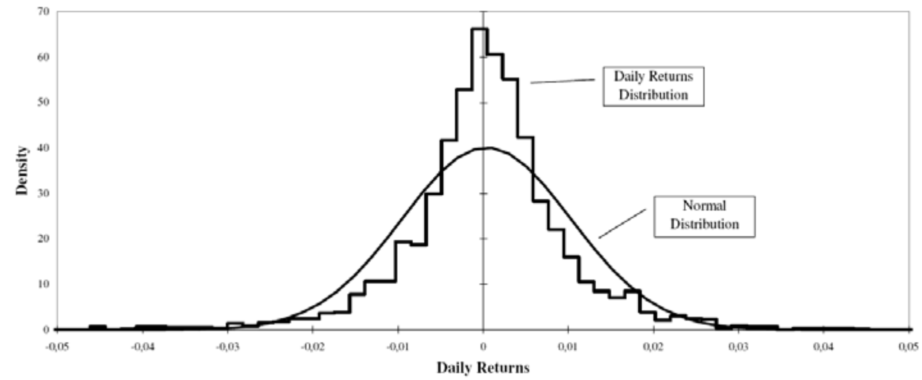
TABLE 1
Descriptive statistics for the returns of the PSI-20 stock index: January 1993 to December 2006

	Daily	Daily (Period 1)	Daily (Period 2)	Daily (Period 3)	Daily (Period 4)	Daily (Period 5)	Weekly	Monthly
Start	01-01-1993	01-01-1993	02-01-1997	23-04-1998	13-03-2000	01-10-2002	01-01-1993	01-01-1993
End	31-12-2006	31-12-1996	22-04-1998	10-03-2000	30-09-2002	31-12-2006	31-12-2006	31-12-2006
Observations	3489	989	322	467	626	1085	729	167
Mean return	0.0004	0.0005	0.0032	0.0001	-0.0017	0.0007	0.0018	0.0080
Annualised return	0.0990	0.1461	1.2074	0.0152	-0.3447	0.1982	0.5738	6.3018
Maximum	0.0694	0.0327	0.0694	0.0540	0.0430	0.0384	0.1212	0.1678
Minimum	-0.0959	-0.0706	-0.0640	-0.0959	-0.0457	-0.0355	-0.1132	-0.2040
St. Deviation	0.0099	0.0068	0.0115	0.0153	0.0121	0.0067	0.0251	0.0569
Skewness	-0.6266	-1.0525	0.0319	-0.8105	-0.2077	-0.0146	-0.2868	-0.3348
Kurtosis	11.0192	17.3398	10.2941	7.9883	4.1061	6.0520	5.8571	4.1283
Jarque-Bera	9576.9**	8656.3**	713.9**	535.3**	36.4**	421.2**	257.9**	12.0**
JB *p*-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0025

Notes: The Jarque-Bera test is a goodness-of-fit measure of departure from normality, based on the sample kurtosis and skewness, and is distributed as a chi-squared with two degrees of freedom. The null hypothesis is a joint hypothesis of both the skewness and excess kurtosis being 0, since samples from a normal distribution have an expected skewness of 0 and an expected excess kurtosis of 0. As the definition of *JB* shows, any deviation from this increases the JB statistic.
* Null hypothesis rejection significant at the 5% level. ** Null hypothesis rejection significant at the 1% level.

The mean returns in the five periods are very different, reflecting the visual criteria used to define those periods. The returns are negatively skewed in almost all periods, and for daily, weekly and monthly data, which means that large negative returns tend to be larger than the higher positive returns. The level of kurtosis is high in the whole sample, but with a tendency to decrease in the later periods. The Jarque-Bera statistic rejects the hypothesis of a normal distribution of returns in all periods and types of data, at a significance level of 1%. The distribution of returns is in fact leptokurtic, as can be confirmed visualy in Figure 2, although it has been approaching the normal distribution in the most recent periods.

FIGURE 2
Distribution of Daily Returns of the PSI-20 stock index: January 1993 to December 2006



Results

Serial Correlation

The results for the tests on serial correlation, Box-Pierce and Ljung-Box statistics are presented in Table 2, for daily, weekly and monthly returns.

TABLE 2
Serial Correlation Coefficients and Q-statistics for Returns of the PSI-20 stock index: January 1993 to December 2006

	Daily	Daily (Period 1)	Daily (Period 2)	Daily (Period 3)	Daily (Period 4)	Daily (Period 5)	Daily (Periods 4 and 5)	Weekly	Monthly
Observations	3479	979	312	457	616	1075	1701	719	157
Lag 1	0.1694**	0.2807**	0.1591**	0.2789**	0.0965*	0.0552	0,0766**	0.0695	0.2143*
Lag 2	-0.0231	0.0380	-0.0413	-0.0578	-0.0915*	0.0283	-0,0361	0.0784*	0.0050
Lag 3	0.0223	-0.0823*	0.0738	-0.0653	0.1338**	-0.0261	0,0821**	0.0322	0.0693
Lag 4	0.0496**	0.0526	-0.0491	0.1558**	-0.0041	0.0982**	0,0465	0.0634	-0.0329
Lag 5	0.0009	0.0027	0.0314	-0.0599	-0.0109	-0.0131	-0,0064	0.0199	-0.0404
Lag 6	-0.0249	-0.0347	-0.0415	-0.0108	-0.0814*	-0.0377	-0,0505*	0.0029	0.0067
Lag 7	0.0294	0.0895**	-0.0596	0.0320	0.0094	0.0607*	0,0287	0.0173	0.0889
Lag 8	0.0400*	-0.0349	0.0549	-0.0039	0.0649	0.0213	0,0635**	0.0051	-0.0443
Lag 9	-0.0288	0.0428	0.0196	-0.0938*	0.0127	-0.0394	-0,0029	0.0592	0.0478
Lag 10	0.0249	0.0704*	-0.0133	0.0350	-0.0155	0.0551	0,0149	-0.0930*	0.0476
Box-Pierce Stat.	127.769**	104.770**	13.952	56.875**	28.977**	26.320**	40,427**	20.780*	10.661
p-value	0.0000	0.0000	0.1752	0.0000	0.0013	0.0033	0,0000	0.0227	0.3845
Ljung-Box Stat.	127.920**	105.238**	14.168	57.441**	29.231**	26.498**	40,568**	20.999*	10.994
p-value	0.0000	0.0000	0.1655	0.0000	0.0011	0.0031	0,0000	0.0211	0.3580

Notes: Both the Box-Pierce statistic and the Ljung-Box statistic test the null hypothesis of overall zero serial correlation coefficients for lags 1 through 10, and are distributed as a chi-square distribution with ten degrees of freedom. For small samples, the Ljung-Box statistic provides a finite-sample correction that yields a better fit to the chi-square distribution.
* Null hypothesis rejection significant at the 5% level. ** Null hypothesis rejection significant at the 1% level.

The daily returns exhibit serial correlation at a significance level of 1% for the total sample and for all the periods, except in period 2, where the *B-P* and *L-B* values are not significant to reject the null hypothesis of zero serial correlation. Some of the lagged variables are significant in one or another period, but the evidence is stronger for lag 4 and, specifically, for lag 1. The regressions strongly prove that the daily return of day *t* is positively correlated with the return of day *t*-1, with a coefficient of around 0.17 for the whole sample 1993 to 2006. One important note is that all the significant coefficients, in all regressions, have a positive sign, thus adding to the global evidence of positive correlation of returns. However, the positive correlation of lag 1 in daily returns has decreased in period 4, and then again in period 5, which may be interpreted has a smaller deviation from the independence of returns inherent in the random walk hypothesis.

The evidence of serial correlation decays as the lag length increases, as it is milder for the weekly data, and for monthly data the overall serial correlation of returns is not significant. This means that the larger the interval of the observations of prices, the less important is the lagged price for explaining future prices. This is consistent with the findings of several other studies including Fama (1965), Panas (1990) and Ma and Barnes (2001). Lastly, we should be cautious in the interpretation of these results, as they assume normality, which we have shown that is not a valid assumption for the distribution of daily returns of the PSI-20 index, in the period 1993 to 2006.

Runs Test

The results of the runs test, which do not depend on normality of returns, are presented in Table 3, for daily, weekly and monthly returns.

TABLE 3
Runs Tests for Daily, Weekly and Monthly Returns of the PSI-20 stock index: January 1993 to December 2006

	Daily	Daily (period 1)	Daily (period 2)	Daily (period 3)	Daily (period 4)	Daily (period 5)	Daily (periods 4 and 5)	Weekly	Weekly (periods 4 and 5)	Monthly	Monthly (periods 4 and 5)
Panel A: positive/negative returns defined relative to zero											
n_+	1836	526	208	226	272	602	875	415	195	96	43
n_-	1653	463	113	240	353	482	835	314	159	71	38
R	1546	413	117	199	292	525	817	324	162	59	28
μ_R	1740.7	493.5	147.4	233.8	308.3	536.4	855.5	358.5	176.2	82.6	41.3
σ_R	29.448	15.652	8.158	10.772	12.280	1 6.253	20.659	13.231	9.297	6.297	4.455
Z	-6.6116**	-5.1426**	-3.7314**	-3.2296**	-1.3234	-0.6988	-1.8652	-2.6077**	-1.5241	-3.7526**	-2.9960**
p-value	0.0000	0.0000	0.0002	0.0012	0.1857	0.4847	0.0622	0.0091	0.1275	0.0002	0.0027
Panel B: positive/negative returns defined relative to the mean return											
n_+	1753	482	162	226	312	546	887	386	202	84	45
n_-	1736	507	159	240	313	538	823	343	152	83	36
R	1546	413	117	199	292	525	817	324	162	59	28
μ_R	1745.5	495.2	161.5	233.8	313.5	543.0	854.8	364.2	174.5	84.5	41.0
σ_R	29.529	15.706	8.943	10.772	12.490	16.454	20.641	13.444	9.206	6.442	4.416
Z	-6.7547**	-5.2326**	-4.9741**	-3.2296**	-1.7213	-1.0922	-1.8314	-2.9926**	-1.3544	-3.9581**	-2.9439**
p-value	0.0000	0.0000	0.0000	0.0012	0.0852	0.2747	0.0670	0.0028	0.1756	0.0001	0.0032

Notes: The runs test tests for a statistically significant difference between the expected number of runs vs. the actual number of runs. A run is defined as sequence of sucessive price changes with the same sign. The null hypothesis is that the successive price changes are independent and random. In Panel A, we define as a positive/negative return any return above/below zero. In Panel B, we define as a positive/negative return any return above/below the mean return.
* Null hypothesis rejection significant at the 5% level. ** Null hypothesis rejection significant at the 1% level.

The number of runs is always less than the expected number of runs, for daily, weekly and monthly data, and for all periods, in line with findings of several international studies (Worthington and Higgs 2004, Abraham *et al*. 2002). This difference is significant at the 1% level for the daily data, for periods 1 to 3 (January 1993 to March 2000). In periods 4 and 5 (March 2000 to December 2006), the number of runs is not statistically different from the expected number of runs, which is consistent with a random walk. The low number of runs in the weekly and monthly returns also refutes the random walk hypothesis, except in the periods 4 and 5, for the weekly data.

Unit Root Tests

In our third test we compute the Augmented Dickey-Fuller statistic to test the null hypothesis of a unit-root in the PSI-20 index prices. We show results is Table 4.

TABLE 4
Augmented Dickey-Fuller tests for the PSI-20
Stock index: January 1993 to December 2006

	Daily	Daily (Period 1)	Daily (Period 2)	Daily (Period 3)	Daily (Period 4)	Daily (Period 5)	Weekly	Monthly
ADF test statistic	-1,3995	-2,0824	1,3256	-0,0597	-2,8460	-1,6406	-1,5521	-1,7205
p-value	0,9990	0,5545	1,0000	0,9954	0,1814	0,7764	0,8107	0,7380
Included observations	3485	979	319	457	616	1074	727	166
Number of lags	4	10	2	9	9	10	2	1

Notes: Augmented Dickey-Fuller statistics test the null hypothesis of a unit root in the stock price series. Failure to reject the null hypothesis means that the random walk hypothesis is not rejected. The number of lags included in the regression is determined by the Akaike Info Criterion.
* Null hypothesis rejection significant at the 5% level. ** Null hypothesis rejection significant at the 1% level.

The number of lagged variables was determined by the Akaike Info Criterion, from a maximum of 10 lags allowed. The results are very clearly in favor of the random walk hypothesis, as the null hypothesis of a unit-root is not rejected for any type of returns (daily, weekly, monthly) or any period. Again, this evidence is consistent with similar findings for the Portuguese stock market, by Dias et al. (2002) and Worthington and Higgs (2004). In any case we have to be cautious about these results, as Liu and He (1991) show that unit root tests may not detect departures from a random walk.

Variance Ratio Tests

Lo and MacKinlay (1988) show that the variance ratio test is more powerful than the Dickey-Fuller unit root test, and Ayadi and Pyun (1994) also argue that the variance ratio has more appealing features than other procedures. Table 5 presents the results of the variance ratios tests for PSI-20 stock index prices. In order to facilitate comparisons with the other studies, we adopt the common procedure of selecting lags 2, 4, 8 and 16.

TABLE 5
Variance Ratio Tests for Lags 2, 4, 8 and 16 for Price Increments of the PSI-20 stock index: January 1993 to December 2006

		Lag 2	Lag 4	Lag 8	Lag 16	Chow-Denning
Daily	$VR(q)$	1,148	1,224	1,346	1,538	
	$Z(q)$	(8,733)**	(7,066)**	(6,905)**	(7,219)**	
	$Z^*(q)$	(3,991)**	(3,329)**	(3,546)**	(3,628)**	(3,991)**
Daily (Period 1)	$VR(q)$	1,266	1,495	1,682	2,086	
	$Z(q)$	(8,375)**	(8,320)**	(7,251)**	(7,761)**	
	$Z^*(q)$	(5,081)**	(5,035)**	(4,643)**	(5,173)**	(5,173)**
Daily (Period 2)	$VR(q)$	1,200	1,291	1,436	1,747	
	$Z(q)$	(3,587)**	(2,786)**	(2,642)**	(3,042)**	
	$Z^*(q)$	(1,724)*	(1,492)	(1,702)*	(2,219)*	(2,219)
Daily (Period 3)	$VR(q)$	1,194	1,258	1,381	1,478	
	$Z(q)$	(4,197)**	(2,977)**	(2,781)**	(2,343)**	
	$Z^*(q)$	(2,957)**	(2,212)*	(2,284)*	(1,899)*	(2,957)*
Daily (Period 4)	$VR(q)$	1,034	0,959	0,934	0,912	
	$Z(q)$	(0,843)	(0,553)	(0,556)	(0,501)	
	$Z^*(q)$	(0,631)	(0,390)	(0,404)	(0,349)	(0,631)
Daily (Period 5)	$VR(q)$	1,050	1,130	1,273	1,474	
	$Z(q)$	(1,656)*	(2,282)*	(3,034)**	(3,546)**	
	$Z^*(q)$	(1,410)	(1,896)*	(2,658)**	(3,068)**	(3,068)**
Daily (Periods 4 and 5)	$VR(q)$	1,051	1,036	1,095	1,212	
	$Z(q)$	(2,110) *	(0,790)	(1,333)	(1,988) *	
	$Z^*(q)$	(1,218)	(0,431)	(0,745)	(1,069)	(1,218)
Weekly	$VR(q)$	1,066	1,225	1,455	1,654	
	$Z(q)$	(1,792)*	(3,250)**	(4,152)**	(4,009)**	
	$Z^*(q)$	(1,084)	(1,894)*	(2,452)**	(2,275)*	(2,452)
Monthly	$VR(q)$	1,226	1,398	1,458	1,807	
	$Z(q)$	(2,921)**	(2,748)**	(2,001)*	(2,368)**	
	$Z^*(q)$	(1,551)	(1,515)	(1,206)	(1,506)	(1,551)

Notes: Variance ratio tests for daily, weekly and monthly PSI-20 index prices. The variance ratios, $VR(q)$, are reported in the first rows, and the variance-ratio test statistics, $Z(q)$ for homoskedastic increments and $Z^*(q)$ for heteroskedastic increments, are reported in parentheses. The null hypothesis is that the variance ratios equal one, which means that the stock index prices follow a random walk. We also show the Chow and Denning (1993) statistic, which tests all the $Z^*(q)$ together,
* Null hypothesis rejection significant at the 5% level. ** Null hypothesis rejection significant at the 1% level.

Except for the daily data in period 4, all variance ratios are larger than unity, which indicates that the variances grow more than proportionally with time. This could be due to heteroskedasticity of stock index prices in some cases, but the $Z^*(q)$ statistic also shows robust results in some of the cases, which is additional proof of autocorrelation in the data. This is consistent with the results of Table 1. The hypothesis of a random walk is rejected by the sample of daily prices for the whole period 1993 to 2006, and the same evidence also applies for the period 1, period 3 and period 5 sub-samples. Period 2 provides mixed evidence, under the assumption of heteroskedasticity which we deem more appropriate, as the variance ratio tests are significant at the 5% level, but the Chow-Denning does not allow the rejection of the null hypothesis of a random walk. In period 4, the random walk is not rejected by any of the tests, and the same is true for periods 4 and 5 together, that is, from March 2000 to December 2006. With weekly data, most of the individual tests reject the null, but the global test provided by the Chow-Denning statistic does not reject a random walk. Monthly data does not reject a random walk behavior.

Conclusions

Table 6 summarizes the results of all the tests performed.

TABLE 6
Summary of Test Results: Random Walk Hypothesis Rejected?

Test	Daily	Daily (Period 1)	Daily (Period 2)	Daily (Period 3)	Daily (Period 4)	Daily (Period 5)	Daily (Periods 4 and 5)	Weekly	Monthly
Serial Correlation Tests									
Lag 1 return significant	*YES*	*YES*	*YES*	*YES*	*YES*	*NO*	*YES*	*NO*	*YES*
B-P and L-B Statistic	*YES*	*YES*	*NO*	*YES*	*YES*	*YES*	*YES*	*YES*	*NO*
Runs Test	*YES*	*YES*	*YES*	*YES*	*NO*	*NO*	*NO*	*YES*	*YES*
Augmented Dickey-Fuller	*NO*	*NO*	*NO*	*NO*	*NO*	*NO*	*NO*	*NO*	*NO*
Variance Ratio Test	*YES*	*YES*	*NO*	*YES*	*NO*	*YES*	*NO*	*NO*	*NO*

Apart from the ADF test, which is very clearly favorable to the random walk hypothesis, all other tests provide mixed evidence. Serial correlation is strong in daily returns, but tends to reduce in weekly data and almost disappears in monthly data. The evidence is more favorable to a random walk in periods 4 and 5, ranging from March 2000 to December 2006. None of the results can be attributed to non-trading or infrequent trading, as the PSI-20 includes only the 20 largest and most liquid shares.

All our findings confirm the previous results on the Portuguese market, namely Dias *et al*. (2002) who find evidence against a random walk until 2001 and Worthington and Higgs (2004) who state that Portugal satisfies the most stringent criteria for a random walk using data until 2003. It seems that, after reaching two bubble-like peaks in April 1998 and March

2000, the Portuguese stock market has become more weak-form efficient in recent years. This is also corroborated by a clear decline of the dependence of daily returns on lagged returns, in the same period.

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