# Can Two-Part Tariffs Promote Efficient Investment on Next Generation Networks?\*

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#### Abstract

We analyze if two-part access tariffs solve the dynamic consistency problem of the regulation of Next Generation Networks. We model the industry as a duopoly, where a vertically integrated incumbent and a downstream entrant, that requires access to the incumbent's network, compete on Hotelling's line. The incumbent can invest in the deployment of a next generation network that improves the quality of the retail services. We have three main results. First, we show that only if the investment cost is low, the regulator can induce investment when he cannot commit to a policy. Second, we show that in this case, two-part tariffs involve payments from the entrant to the incumbent that may be politically unacceptably high. Third, we show that if the regulator can commit to a policy, a regulatory moratorium may emerge as socially optimal.

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### 1 Introduction

The deployment of next generation networks, leading to multi-service networks for audio, video, and data services, sets the telecommunications sector on the verge of a new era.<sup>1</sup> In order to give firms the right incentives to invest, and to promote an efficient use of these infrastructures, sectoral regulators must set an adequate regulatory framework for these new telecommunications networks.

There is at least one important difference between the regulation of current and the next generation networks. The former are already deployed, whereas the latter are not.<sup>2</sup> This implies that the regulatory policy should balance the conflicting goals of reducing the incumbent's market power, namely on the wholesale market, and giving the incumbent incentives to invest in a next generation network. In other words, the regulatory policy should trade-off static and dynamic efficiency.<sup>3</sup>

This trade-off may generate a dynamic consistency problem. Before the network is deployed, it is socially optimal to set high access tariffs to promote investment. However, once the network is deployed, it is socially optimal to set the access tariff to promote competition in the retail market. This dynamic consistency problem affects investment negatively. The incumbent anticipates that it will be expropriated from the incremental profit of its investment, and reduces investment.

It is possible, in principle, for either the regulator or the legislator, to adopt measures that constrain the regulator's future actions.<sup>4</sup> However, this is only feasible for short periods, while the investment cycle for telecommunications networks is very long. Therefore, in practice, it will be hard for the regulators to credibly commit to a regulatory policy towards next generation networks.

In this article, we analyze if two-part tariffs can solve the dynamic consistency problem

<sup>&</sup>lt;sup>1</sup>A Next Generation Network is a "(...) packet-based network able to provide telecommunication services and able to make use of multiple broadband, QoS-enabled transport technologies and in which service-related functions are independent from underlying transport related technologies." See ITU (2001).

 $<sup>^{2}</sup>$ Besides, the investment in the current networks occurred while the industry was a legal monopoly.

<sup>&</sup>lt;sup>3</sup>According to ERG (2007), "welfare gains can result from two main sources: Static efficiency gains (derived from the most efficient use of existing technologies. Static efficiency is maximised through intense competition and subsequent lower prices), and dynamic efficiency gains (gains related to the additional value generated by innovative new technologies and services that may be produced at lower cost and customers may attach a higher value to)".

<sup>&</sup>lt;sup>4</sup>Guthrie (2006) discusses the constraints on the regulator's actions adopted in several countries to prevent the regulator from acting opportunistically. For instance, the regulator can announce that he will set the access price at a certain level until the next scheduled review.

of the regulation of next generation networks. With two-part access tariffs, compared with linear access tariffs, the regulator gains an additional instrument, the fixed fee, which is neutral in terms of welfare. This might enable the regulator to give the incumbent incentives to invest, even when he cannot commit to a regulatory policy.

We model the industry as a differentiated products duopoly, where an incumbent and an entrant compete on Hotelling's line (Hotelling, 1929). The incumbent is a vertically integrated firm that owns a network, and operates on the retail market. The entrant operates on the retail market, and requires access to the incumbent's network. The incumbent can invest in the deployment of a next generation network that improves the quality of the retail services. The sectoral regulator sets the access tariff to the incumbent's network.

If the investment cost is low, the regulator can set the marginal price of the access tariffs at marginal cost, and use the fixed fee to give the incumbent incentives to invest. Since the fixed fee is neutral in terms of welfare, the regulatory policy is dynamically consistent. However, this solution involves setting the fixed part of the access tariffs at a level that might be politically unacceptably high. Moreover, when the regulator cannot commit to a policy, this is only one of two types of equilibria, and in the other type of equilibria the incumbent does not invest, although investing would increase welfare.

If the investment cost is high, the fixed fee is no longer enough to induce investment. The regulator has to raise the marginal price of the access tariff above marginal cost to induce investment. Since, after investment occurs, it is socially optimal to set the marginal price of the access tariff at marginal cost, this policy is not dynamically consistent.

Interestingly, a regulatory moratorium emerges as socially optimal, if the regulator can commit to a policy, and if the investment cost takes intermediate values.

The academic literature on regulation only recently started to address the relation between access pricing and investment. Guthrie (2006) surveys the recent literature on the relationship between infrastructure investment and the different regulatory regimes. He concludes that much remains to be done. Valletti (2003) argues that it is important that regulatory policies are designed in a way that enables regulators to commit to rules over a reasonable time period. Regulators should try to stabilize their policies to signal to firms that they can commit to their decisions.

Gans (2001) is the article closest to ours. He analyzes an investment timing game, where two firms compete to build a new infrastructure. His results are substantially different from ours. In his model there is never a dynamic consistency problem. This is a consequence of the preemption effect, typical of the investment race literature. Besides, in his model the access tariff is such that the investment costs are fully distributed, whereas in ours the entrant must pay a considerable part of the investment cost. Gans and King (2004) study the impact of access regulation on the timing of infrastructure investment, when there is uncertainty about the investment returns. This article suggests the use of a regulatory moratorium when the regulator has commitment problems. Vareda and Hoernig (2007) study the investment of two operators in new infrastructures, which allows them to offer new services, and show that a regulatory moratorium may be a necessary tool to give the leader the correct incentives to invest, at the same time that allows to charge a lower access price later on in order to delay the follower's investment. Vareda (2007) studies the incumbent's incentives to invest in quality upgrades and cost reduction when the regulator forces it to unbundle its network. Foros (2004) shows that under some conditions the investment by an incumbent in the quality of its network is lower with price regulation since the access price is set equal to marginal cost. Kotakorpi (2006) considers a similar model with vertical differentiation, and obtains similar results. Brito et all. (2008) analyze the case where access to next generation networks is not regulated, and shows that in case of a non-drastic innovation the incumbent will prefer to give access to the next generation network if it is forced by the regulator to give access to the old network at a low access price. Caillaud and Tirole (2004) analyze the funding of an infrastructure when an incumbent has private information about the profitability of the investment and the regulator does not have access to taxpayers' money.

The remainder of the article is organized as follows. We describe the model in Section 2. In Sections 3 and 4, we analyze the commitment and no-commitment games, respectively. In Section 5, we compare the two games and discuss policy implications. Finally, in Section 6, we conclude. All proofs are in the Appendix.

### 2 Model

#### 2.1 Environment

Consider a telecommunications industry where two firms, the incumbent and the entrant, sell horizontally differentiated products. The *incumbent*, firm i, is a vertically integrated firm that owns a bottleneck input, to which we refer to as the old network. The *old network*, network o, is a telephone network with a local access network based on the twisted pair of copper wire. The incumbent can make an investment to deploy a next generation network. The next generation network is also a bottleneck input that allows the supply of retail products of a higher quality than those supplied through the old network. We refer to the next generation network as the *new network*, or network *n*. The *entrant*, firm *e*, only operates in the retail market, and has to buy access to the incumbent's network. We index firms with subscript j = i, e, and networks with subscript v = o, n.

There is a third party in the industry, the sectoral regulator.

Costs and demand are common knowledge.

#### 2.2 Consumers

There is a large number of consumers, formally a continuum, whose measure we normalize to 1. Consumers are uniformly distributed along a Hotelling line segment of length 1, facing transportation costs tx to travel distance x, with t on  $[0, +\infty)$ . Consumers are otherwise homogeneous. As in Biglaiser and DeGraba (2001), we assume that each consumer has a demand function for telecommunication services given by  $y_j = (z + \Delta_v) - p_j$ , where  $y_j$  on  $(0, z + \Delta_v)$  is the number of units of telecommunication services purchased from firm j,  $p_j$ on  $(0, z + \Delta_v)$  is the per unit price of telecommunication services of firm j, z is a parameter on  $(\frac{4}{3}\sqrt{6t}, +\infty)$ , and  $\Delta_v$  is a parameter that takes value 0 for products supplied through the old network and takes value  $\Delta_z$  on  $(0, +\infty)$  for products supplied through the new network.<sup>5</sup> This means that consumers are willing to pay a premium for services delivered through the new network. The lower limit on z implies that all consumers have a positive surplus under the different market structures.

Let  $\chi := \Delta_z (2z + \Delta_z)$ . For  $p_j = 0$ , the incremental consumer surplus from the investment is  $\frac{1}{2}\chi$ . We take  $\chi$  as a measure of quality improvement enabled by the next generation network.

#### 2.3 Sectoral Regulator

The regulator sets the wholesale tariff the entrant must pay to have access to the incumbent's network.<sup>6</sup> Denote by  $A_v(y_e) = K_v + \alpha_v y_e$ , the access tariff to network v = o, n, where  $\alpha_v$  on  $[0, +\infty)$  is the per unit price of telecommunication services, or the marginal access price, and  $K_v$  on  $[0, \frac{1}{2}t)$  is the fixed access fee.<sup>7</sup> The upper limit on  $K_v$  implies that

<sup>&</sup>lt;sup>5</sup>Units of telecommunication services could be, e.g., minutes of communication or megabits.

<sup>&</sup>lt;sup>6</sup>Regulating telecommunications markets by intervening at the wholesale level, namely by setting access prices, corresponds to the current EU and US practice.

<sup>&</sup>lt;sup>7</sup>If the investment represents an upgrade of the existing network, instead of the deployment of a different network, the access tariff to the new network can be interpreted as the price paid by the entrant to have access to higher quality wholesale services. The fixed part is independent of the number of minutes and of

if the regulator wants to create a monopoly on one of the networks, he must set a very high marginal access price.

The regulator maximizes social welfare, i.e., the sum of the firms' profit and the consumer surplus, denoted by W.

#### 2.4 Firms

The incumbent produces an input that: (i) uses in the production of a retail product, or (ii) sells to the entrant.

All of the incumbent's marginal costs are constant and equal to zero. The entrant has marginal costs  $\alpha_v$  on  $\{\alpha_o, \alpha_n\}$ , if it uses network v = o, n.

The incumbent is located at point 0 and the entrant at point 1 of the line segment where consumers are distributed.

Firms charge consumers two-part *retail tariffs*, denoted by  $T_j(y_j) = F_j + p_j y_j$ , j = i, e, where  $F_j$  on  $[0, +\infty)$  is the fee of firm j.

At a cost I on  $(0, \overline{I})$ , where  $\overline{I} := \frac{1}{2} [(z + \Delta_z)^2 - 3t]$ , the incumbent can deploy a new network.<sup>8</sup> The upper limit on I ensures that the incumbent invests if this allows it to move from a duopoly on the old network with  $K_o = \alpha_o = 0$ , to a monopoly on the new network. To simplify the exposition, we assume that the old network is phased out when the new network is deployed.

Denote by  $D_j$ , the demand, in terms of consumers, for firm j = i, e. Under duopoly, the profits of firm j = i, e for the whole game are:<sup>9</sup>

$$\pi_i = \left[p_i \left(z + \Delta_v - p_i\right) + F_i\right] D_i + K_v + \alpha_v \left(z + \Delta_v - p_e\right) D_e - \frac{\Delta_v}{\Delta_z} I,$$
$$\pi_e = \left[\left(p_e - \alpha_v\right) \left(z + \Delta_v - p_e\right) + F_e\right] D_e - K_v.$$

the number of consumers.

<sup>&</sup>lt;sup>8</sup>We assume that the investment cost of the entrant is larger than the investment cost of the incumbent, and too high for the investment to be profitable, i.e., the investment cost of the entrant belongs to  $\left(\frac{1}{2}\left(z+\Delta_d\right)^2-t,+\infty\right)$ . This happens because the entrant has to build a network from scratch, while the incumbent just needs to upgrade its old network. Alternatively, the entrant has a higher cost of capital. WIK (2008) shows that it is 30% cheaper for incumbents to roll-out fibre networks than for entrants.

<sup>&</sup>lt;sup>9</sup>For each consumer served by the entrant the incumbent earns  $\alpha_v (z + \Delta_v - p_e)$ , i.e., the wholesale markup times the number of minutes sold to each consumer. This represents the opportunity cost for the incumbent of serving directly each consumer.

#### 2.5 Timing of the Game

We consider two games. In the *commitment game*, the sectoral regulator can commit to a regulation policy towards the new network before the investment is made; in the *nocommitment game*, he cannot.

The *commitment game* has four stages which unfold as follows. In stage 1, the sectoral regulator sets the access tariffs to the old and the new networks. In stage 2, the incumbent decides whether to invest. In stage 3, the entrant decides if it stays in the market or exits. In stage 4, the incumbent and the entrant compete on retail tariffs.

The no-commitment game has five stages, which unfold as follows. In stage 1, the regulator sets the access tariff to the old network. In stage 2, the incumbent decides whether to invest. In stage 3, the sectoral regulator sets the access tariff to the new network. In stage 4, the entrant decides if it stays in the market or exits. In stage 5, the incumbent and the entrant compete on retail tariffs.

These games represent two polar cases. In practice, the regulator has some ability to commit to a policy, particularly for a short period, but cannot commit completely to a policy, particularly for a long period. Thus, the critical issue is whether the regulator can commit to a regulatory policy for a period as long as the investment cycle of the new network.

#### 2.6 Equilibrium Concept

The sub-game perfect Nash equilibrium for the commitment game is: (i) a pair of access tariffs, (ii) an investment decision, (iii) a decision of whether to stay in the market, (iv) a pair of retail tariffs, such that:

(E1) the retail tariffs maximize the firms' profits, given the access tariffs, the investment decision and the exit decision;

(E2) the decision to exit the market maximizes the entrant's profits, given the access tariffs, the incumbent's investment decision, and the optimal retail tariffs function;

(E3) the investment decision maximizes the incumbent's profits, given the access tariffs, the optimal entry decision and the optimal retail tariffs function;

(E4) the access tariffs for the old and new networks maximize social welfare, given the optimal investment decision, the optimal entry decision, and the optimal retail tariffs function.

Similarly for the no-commitment game.

### 3 Equilibrium of the Commitment Game

In this Section, we characterize the equilibria of the commitment game, which we construct by working backwards.

#### 3.1 Retail Price Game

We characterize the equilibria of the retail price game for four cases: (i) the incumbent does not invest in the new network, and the entrant exits the market, (ii) the incumbent invests in the new network, and the entrant exits the market, (iii) the incumbent does not invest in the new network, and the entrant stays in the market, (iv) the incumbent invests in the new network, and the entrant stays in the market.<sup>10</sup> In cases (i)-(ii) the retail market is a monopoly. In cases (iii)-(iv) the retail market is a duopoly. We use superscripts  $m_o$ ,  $m_n$ ,  $d_o$ ,  $d_n$  to denote variables or functions associated with cases (i)-(iv), respectively. In what follows we use the expression "net" as a shorthand for "net of the investment cost".

We start with the following Lemma.

**Lemma 1:** In equilibrium, firms set the marginal price of the two-part retail tariff at marginal cost, i.e.,  $p_i = 0$  and  $p_e = \alpha_v$ , for v = o, n.

As usual with two-part tariffs, firms set the variable part of the retail tariff at marginal cost, to maximize gross consumer surplus, and then try to extract this surplus using the fixed fee.

Given Lemma 1, from now on we only discuss the determination of the fixed fees.

#### 3.1.1 Monopoly

Next, we characterize the equilibrium of the retail price game for the two cases where the retail market is a monopoly, which are presented in the next Lemma.

**Lemma 2:** If the retail market is a monopoly, in equilibrium, the incumbent charges the fixed fee, for v = n, o:

$$F_i^{m_v}(\Delta_v) = \frac{\left(z + \Delta_v\right)^2}{2} - t$$

<sup>&</sup>lt;sup>10</sup>A duopoly where the incumbent uses the new network and the entrant uses the old network is impossible. By assumption, the old network is phased out once the new network is deployed.

The net profit of the incumbent for v = n, o, is:

$$\pi_i^{m_v}\left(\Delta_v, I\right) = \frac{1}{2} \left(z + \Delta_v\right)^2 - t - \frac{\Delta_v}{\Delta_z} I.$$

#### 3.1.2 Duopoly

Next, we characterize the equilibrium of the retail price game for the two cases where the retail market is a duopoly, which are presented in the next Lemma.

**Lemma 3:** If the retail market is a duopoly, in equilibrium, the incumbent and the entrant charge the fixed fees, for v = n, o:

$$F_{i}^{d_{v}}(\Delta_{v},\alpha_{v}) = \begin{cases} t + \frac{1}{6}\alpha_{v}\left[6\left(z + \Delta_{v}\right) - 5\alpha_{v}\right] & \text{for } \alpha_{v} \text{ on } \left[0,\sqrt{6t}\right) \\ \alpha_{v}\left(z + \Delta_{v}\right) - \frac{1}{2}\alpha_{v}^{2} - t & \text{for } \alpha_{v} \text{ on } \left[\sqrt{6t}, z + \Delta_{v}\right] \end{cases}$$
$$F_{e}^{d_{v}}(\Delta_{v},\alpha_{v}) = \begin{cases} t - \frac{1}{6}\alpha_{v}^{2} & \text{for } \alpha_{v} \text{ on } \left[0,\sqrt{6t}\right) \\ 0 & \text{for } \alpha_{v} \text{ on } \left[\sqrt{6t}, z + \Delta_{v}\right]. \end{cases}$$

The net profits of the incumbent and the entrant, gross of the fixed fee of the access tariff, for v = n, o are, respectively:<sup>11</sup>

$$\pi_i^{d_v}\left(\Delta_v, \alpha_v; I\right) = \begin{cases} \frac{\left(36t^2 + \alpha_v^4 - 60t\alpha_v^2\right) + 72\alpha_v t(z + \Delta_v)}{72t} - \frac{\Delta_v}{\Delta_z}I & \text{for } \alpha_v \text{ on } \left[0, \sqrt{6t}\right)\\ \alpha_v\left(z + \Delta_v\right) - \frac{1}{2}\alpha_v^2 - t - \frac{\Delta_v}{\Delta_z}I & \text{for } \alpha_v \text{ on } \left[\sqrt{6t}, z + \Delta_v\right], \end{cases}$$

and

$$\pi_e^{d_v} \left( \Delta_v, \alpha_v \right) = \begin{cases} \frac{\left(6t - \alpha_v^2\right)^2}{72t} & \text{for } \alpha_v \text{ on } \left[0, \sqrt{6t}\right) \\ 0 & \text{for } \alpha_v \text{ on } \left[\sqrt{6t}, z + \Delta_v\right]. \end{cases}$$

In a duopoly, the profit of the incumbent is non-decreasing in the marginal access price, while the profit of the entrant is non-increasing in the marginal access price.<sup>12</sup> When the marginal access price increases, the marginal cost of the entrant increases relative to that of the incumbent. As a consequence, the market share, and thereby the profit of the incumbent, increases, while the entrant's profit decreases.

#### **3.2** Exit Decision

Next, we characterize the entrant's optimal decision of whether to stay or exit the market.

<sup>&</sup>lt;sup>11</sup>For  $\alpha_v$  on  $[z + \Delta_v, +\infty)$ , the incumbent's profit equals the monopoly profit defined in section 3.1.1.

<sup>&</sup>lt;sup>12</sup>The first part follows from the assumption that z belongs to  $\left(\frac{4}{3}\sqrt{6t}, +\infty\right)$ .

When indifferent between staying or exiting the market, the entrant chooses the latter. Let the incumbent own network v, and  $\alpha_v$  be on  $[0, \sqrt{6t}]$ . Then, the entrant:

stays in the market if 
$$\frac{(6t-\alpha_v^2)^2}{72t} - K_v > 0$$
  
exits if  $\frac{(6t-\alpha_v^2)^2}{72t} - K_v \le 0$ .

For  $\alpha_v$  on  $\left[\sqrt{6t}, z + \Delta_v\right]$  the entrant exits the market.

#### **3.3** Investment Decision

Next, we characterize the incumbent's optimal investment decision.

Denote by  $\Delta \Pi^c(\Delta_z, K_n, \alpha_n, K_o, \alpha_o; I)$ , the net incremental profit of the investment for the commitment game.<sup>13</sup>

The incumbent invests in the new network, if and only if,  $\Delta \Pi^{c}(\Delta_{z}, K_{n}, \alpha_{n}, K_{o}, \alpha_{o}; I) > 0.$ 

Depending on the access tariffs for both networks and the investment cost, the investment may or may not occur. If the investment cost is too high, there is no-investment, whatever the levels of the access tariffs. However, if the investment cost is not too high, by choosing the access tariffs, the regulator can influence the outcome of the investment decision.

#### **3.4** Regulation of the New and Old Networks

Next, we characterize the socially optimal access tariff to the new and old networks. For v = n, o, the regulator's objective function in case of a monopoly is given by:

$$W^{m_v}\left(\Delta_v,I\right) = \frac{1}{2}\left[\left(z+\Delta_v\right)^2 - t\right] - \frac{\Delta_v}{\Delta_z}I;$$

and in the case of duopoly is given by:

$$W^{d_v}\left(\Delta_v, \alpha_v; I\right) = \begin{cases} \frac{72t(z+\Delta_v)^2 + 5\alpha_v^4 - 36t\left(t+\alpha_v^2\right)}{144t} - \frac{\Delta_v}{\Delta_z}I & \text{for } \alpha_v \text{ on } \left[0, \sqrt{6t}\right)\\ \frac{(z+\Delta_v)^2 - t}{2} - \frac{\Delta_v}{\Delta_z}I & \text{for } \alpha_v \text{ on } \left[\sqrt{6t}, z+\Delta_v\right]. \end{cases}$$

$$[Figure 1]$$

Figure 1 illustrates the welfare function,  $W^{d_v}(\cdot)$ .<sup>14</sup> Function  $W^{d_v}(\cdot)$  is quasi-convex in  $\alpha_v$  because increasing  $\alpha_v$  has the following three effects. First, it has the negative effect of increasing transportation costs. Second, it has the negative effect of leading the entrant

 $<sup>^{13}\</sup>mathrm{See}$  the definition in the Appendix.

<sup>&</sup>lt;sup>14</sup>The welfare function,  $W^{d_v}(\cdot)$  is quasi-convex in  $\alpha_v$ ; it is decreasing for  $\alpha_v$  on  $\left[0, \frac{3}{5}\sqrt{10t}\right]$ , and increasing for  $\alpha_v$  on  $\left[\frac{3}{5}\sqrt{10t}, \sqrt{6t}\right]$ . Additionally,  $W^{d_v}\left(\Delta_v, \sqrt{\frac{6t}{5}}; I\right) = W^{d_v}\left(\Delta_v, \sqrt{6t}; I\right) = W^{m_v}\left(\Delta_v, I\right)$ , and  $W^{d_v}\left(\Delta_v, 0; I\right) > W^{m_v}\left(\Delta_v, I\right)$ .

to set a higher marginal retail price. Third, it has the positive effect of shifting some consumers from the entrant, where they have face a higher marginal retail price, to the incumbent, where they face a lower marginal retail price. If the access price is zero, the third effect is absent because the marginal price set by both firms is equal. Thus, increasing  $\alpha_v$  unambiguously lowers welfare. If  $\alpha_v$  is sufficiently high, the third effect may more than compensate the other two.

For a given  $\alpha$ , investment may shift the welfare function upwards or downwards. This occurs because

$$W^{d_v}\left(\Delta_z, \alpha; I\right) - W^{d_v}\left(0, \alpha; I\right) = \frac{\chi}{2} - I,$$

may be positive or negative. However, for investment to occur the regulator may need to change the value of  $\alpha$ .

Thus, the regulator's choice of the two-part tariffs for both networks impacts welfare directly and indirectly. Welfare is impacted directly because the marginal access prices affect: the retail marginal prices, the amount consumers purchase, and the transportation costs. Welfare is impacted indirectly because the choice of the two-part tariffs parameters may change the incumbent's incentives to invest. The regulator's choice is thus between the socially optimal two-part tariffs that lead to no-investment, and the socially optimal two-part tariffs that leads to investment.

Let  $\Delta_K := K_n - K_o$ . When  $(\alpha_o, \alpha_n) = (0, 0)$ ,  $\Delta_K$  can be interpreted as the incremental profit of the investment, since in this case profit is invariant to  $\Delta_v$ .

If the regulator wants to induce no-investment, he optimally sets  $\alpha_o = \alpha_n = 0$  and  $\Delta_K$ such that the net incremental profit of the investment is negative, i.e., sets  $\Delta_K < I$ .

If the regulator wants to induce investment, the choice is less straightforward because the regulator will choose  $(K_n, K_o, \alpha_n, \alpha_o)$  to maximize welfare, subject to the constraint that the incumbent invests, i.e., subject to the constraint  $\Delta \Pi^c(\Delta_z, K_n, K_o, \alpha_n, \alpha_o; I) > 0$ .

With this purpose, he sets  $(K_o, \alpha_o) = (0, 0)$ , so that the profit without investment is as low as possible, and sets, at most,  $\Delta_K = K_n = \pi_e^{d_n} (\Delta_z, \alpha_n) - \varepsilon$ , with  $\varepsilon \to 0^+$ , thus relaxing the constraint. Thus, the incumbent's profit increases with investment from the individual duopoly profit with access price set at marginal cost to the industry aggregate profit. If  $\alpha_n$ is on  $[0, \sqrt{6t})$  the industry profit corresponds to the sum of both duopolists profit at access price  $\alpha_n$ . The incumbent receives the entrant's profit through the fixed fee. If  $\alpha_n$  is on  $[\sqrt{6t}, z + \Delta_z)$  the industry profit corresponds to the monopoly profit because the entrant will exit the market. The regulator should set  $\alpha_n = 0$  if, at this level, the constraint is not binding. In this case, the fixed fee need not be equal to the entrant's profit. However, if the constraint is binding, even with the fixed fee equal to the entrant's profit, a higher access price is called for.<sup>15</sup>

Denote by  $\hat{\alpha}_n(I)$ , the socially optimal marginal access price that leads the incumbent to invest. Due to the quasi-convexity of the welfare function,  $\hat{\alpha}_n(I)$  is either the minimum or the maximum of the set of marginal prices of the access tariffs for which investment is profitable. In the appendix, we show that:

$$\widehat{\alpha}_n(I) = \begin{cases} 0 & \text{for } I \text{ on } \left[0, \frac{1}{2}t\right] \\ b_n(I) & \text{for } I \text{ on } \left(\frac{1}{2}t, \sqrt{\frac{6}{5}t}\left(z + \Delta_z\right) - \frac{33}{50}t\right] \\ \sqrt{6t} & \text{for } I \text{ on } \left(\sqrt{\frac{6}{5}t}\left(z + \Delta_z\right) - \frac{33}{50}t, +\infty\right), \end{cases}$$

where  $b_n(I)$  is defined by  $\frac{1}{2}t + \frac{b_n[36t(z+\Delta_z-b_n)+b_n^3])}{36t} - I \equiv 0.$ 

Denote by I, the investment cost level for which the net incremental welfare benefit of the investment is 0 under duopoly if the regulator sets  $\alpha_n$  optimally, i.e.,  $W^{d_n}\left(\Delta_z, \widehat{\alpha}_n(\widetilde{I}); \widetilde{I}\right) - W^{d_o}(0,0;0) \equiv 0$ . Let  $\chi^c := \left(\frac{2\sqrt{15}}{25}\sqrt{50\frac{z^2}{t}+19} + \frac{79}{50}\right)t$ .

The next Lemma presents the equilibrium access tariffs for the old and new networks.

Lemma 4: For the commitment game, there are four types of socially optimal access tariffs. (i) If  $(\chi, I)$  is on  $[0, t) \times (0, \frac{1}{2}\chi)$ , or, if  $(\chi, I)$  is on  $[t, +\infty) \times (0, \frac{1}{2}t)$ , then the regulator sets access tariffs  $(\Delta_K^*, \alpha_o^*, \alpha_n^*)$  on  $(I, \frac{1}{2}t) \times \{0\} \times \{0\}$ . (ii) If  $(\chi, I)$  is on  $[t, \chi^c) \times [\frac{1}{2}t, \widetilde{I})$ , or if  $(\chi, I)$  is on  $[\chi^c, +\infty) \times [\frac{1}{2}t, \sqrt{\frac{6}{5}t}(z + \Delta_z) - \frac{33}{50}t)$ , then the regulator sets access tariffs  $(\Delta_K^*, \alpha_o^*, \alpha_n^*)$  on  $(I - \pi_i^{d_n}(\Delta_z, b_n(I)) + \frac{1}{2}t, \pi_e^{d_n}(\Delta_z, b_n(I))) \times \{0\} \times \{b_n(I)\}$ . (iii) If  $(\chi, I)$  is on  $[\chi^c, +\infty) \times [\sqrt{\frac{6}{5}t}(z + \Delta_z) - \frac{33}{50}t, \frac{1}{2}x - \frac{1}{4}t)$ , then the regulator sets access tariffs  $(\Lambda_i^{m_n}(\Delta_z) - I - \frac{1}{2}t) \times [0, \frac{1}{2}t) \times \{0\} \times \{\sqrt{6t}\}$ . (iv) If  $(\chi, I)$  is on  $[0, t) \times [\frac{1}{2}\chi, \overline{I})$ , or, if  $(\chi, I)$  is on  $[t, \chi^c) \times [\widetilde{I}, \overline{I})$ , or, if  $(\chi, I)$  is on  $[\chi^c, +\infty) \times [\frac{1}{2}\chi - \frac{1}{4}t, \overline{I})$ , then the regulator sets access tariffs  $(\Delta_K^*, \alpha_o^*, \alpha_n^*)$  on  $[0, I] \times \{0\} \times \{0\}$ .

Given  $\chi$ , I increases as one moves from case (i) to case (iv). The value of  $\alpha_n^*$  increases from 0, in case (i), to  $\sqrt{6t}$  in case (iii), and then falls back to 0 in case (iv). We give a detailed explanation for this result in the next section, which summarizes the equilibria of the whole game.

<sup>&</sup>lt;sup>15</sup>The sum of the incumbent and the entrant's profits is increasing in the access price.

#### 3.5 Equilibrium of the Whole Game

Having solved all the four stages of the commitment game, we now summarize the equilibria of the whole game, which we present in the next Proposition for further reference. For clarity of exposition, we divide this Proposition in three cases, which we present in turn: (i) small increase in quality:  $\chi$  is on [0, t); (ii) intermediate increase in quality:  $\chi$  is on  $[t, \chi^c)$ ; and (iii) large increase in quality:  $\chi$  is on  $[\chi^c, +\infty)$ .

**Proposition 1a:** If the increase in quality is small, i.e., if  $\chi$  is on [0, t), the commitment game has two types of equilibria:

(I) If I is on  $(0, \frac{1}{2}\chi)$ : (i) the regulator sets access tariffs  $(\Delta_K^*, \alpha_o^*, \alpha_n^*)$  on  $(I, \frac{1}{2}t) \times \{0\} \times \{0\}$ , (ii) the incumbent invests in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs  $F_i^* = t$ ,  $F_e^* = t$ , and  $p_i^* = 0$ ,  $p_e^* = 0$ .

(II) If I is on  $\left[\frac{1}{2}\chi,\overline{I}\right)$ : (i) the regulator sets access tariffs  $(\Delta_K^*, \alpha_o^*, \alpha_n^*)$  on  $[0, I] \times \{0\} \times \{0\}$ , (ii) the incumbent does not invest in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs  $F_i^* = t$ ,  $F_e^* = t$ , and  $p_i^* = 0$ ,  $p_e^* = 0$ .

The social desirability of investment depends on  $\chi$ , the measure of the quality improvement enabled by the new network, and the investment cost, I. If the increase in quality is small, investment is only socially desirable for low values of the investment cost, i.e., for I on  $(0, \frac{1}{2}\chi)$ . For these low values of I, the regulator can set the marginal access price to marginal cost,  $\alpha_n = 0$ , and use the fixed access fee to induce investment. The fixed access fee will, at most, equal the entrant's profit. However, this is enough to cover low values of the investment cost, i.e.,  $\Delta_K \geq I$ , without the need to increase the access price above marginal cost.

We now turn to the case of an intermediate increase in quality.

**Proposition 1b:** If the increase in quality takes intermediate values, i.e., if  $\chi$  is on  $[t, \chi^c)$ , the commitment game has three types of equilibria:

(I) If I is on  $(0, \frac{1}{2}t)$ : (i) the regulator sets access tariffs  $(\Delta_K^*, \alpha_o^*, \alpha_n^*)$  on  $(I, \frac{1}{2}t) \times \{0\} \times \{0\}$ , (ii) the incumbent invests in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs  $F_i^* = t$ ,  $F_e^* = t$ , and  $p_i^* = 0$ ,  $p_e^* = 0$ .

(II) If I is on  $\left\lfloor \frac{1}{2}t, \widetilde{I} \right\rfloor$ : (i) the regulator sets access tariffs  $(\Delta_K^*, \alpha_o^*, \alpha_n^*)$  on  $\left(I - \pi_i^{d_n}(\Delta_z, b_n(I)) + \frac{1}{2}t, \pi_e^{d_n}(\Delta_z, b_n(I))\right) \times \{0\} \times \{b_n(I)\}$ , (ii) the incumbent invests in the new technology,

(iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs  $F_i^* = F_i^{d_n}(\Delta_z, b_n(I)), \ F_e^* = F_e^{d_n}(\Delta_z, b_n(I)), \ \text{and} \ p_i^* = 0, \ p_e^* = b_n(I).$ 

(III) If I is on  $[\tilde{I}, \bar{I}]$ : (i) the regulator sets access tariffs  $(\Delta_K^*, \alpha_o^*, \alpha_n^*)$  on  $[0, I] \times \{0\} \times \{0\}$ , (ii) the incumbent does not invest in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs  $F_i^* = t$ ,  $F_e^* = t$ , and  $p_i^* = 0$ ,  $p_e^* = 0$ .

Now since  $\chi$  is larger, compared with the case of Proposition 1a, investment is socially desirable for higher values of the investment cost, i.e., for I on  $(0, \tilde{I})$ .

As with Proposition 1a, if I is small, i.e., if I is on  $(0, \frac{1}{2}t)$ , the regulator can set the marginal access price at marginal cost, and use the fixed access fee to induce investment. If I takes higher values, i.e., if I is on  $(\frac{1}{2}t, \tilde{I})$ , transferring all the entrant's profit to the incumbent through the fixed fee is not enough to induce investment. The regulator also has to raise the marginal access price above the marginal cost. This distorts competition in the retail market. If I takes even higher values, i.e., if I is on  $(\tilde{I}, \bar{I})$ , the regulator induces no-investment. With the marginal access price at marginal cost, investment increases welfare for I on  $(\frac{1}{2}\chi, \tilde{I})$ . However, given the distortions created by raising the marginal access price above than the investment cost.

We now turn to the case of a large increase in quality.

**Proposition 1c:** If the increase in quality is large, i.e., if  $\chi$  is on  $[\chi^c, +\infty)$ , the commitment game has four types of equilibria:

(I) If I is on  $(0, \frac{1}{2}t)$ : (i) the regulator sets access tariffs  $(\Delta_K^*, \alpha_o^*, \alpha_n^*)$  on  $(I, \frac{1}{2}t) \times \{0\} \times \{0\}$ , (ii) the incumbent invests in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs  $F_i^* = t$ ,  $F_e^* = t$ , and  $p_i^* = 0$ ,  $p_e^* = 0$ .

(II) If I is on  $\left[\frac{1}{2}t, \sqrt{\frac{6}{5}t}\left(z+\Delta_z\right)-\frac{33}{50}t\right)$ : (i) the regulator sets access tariffs  $(\Delta_K^*, \alpha_o^*, \alpha_n^*)$ on  $\left(I-\pi_i^{d_n}\left(\Delta_z, b_n(I)\right)+\frac{1}{2}t, \pi_e^{d_n}\left(\Delta_z, b_n(I)\right)\right) \times \{0\} \times \{b_n(I)\}$ , (ii) the incumbent invests in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs  $F_i^* = F_i^{d_n}\left(\Delta_z, b_n(I)\right), F_e^* = F_e^{d_n}\left(\Delta_z, b_n(I)\right)$ , and  $p_i^* = 0, p_e^* = b_n(I)$ .

(III) If I is on  $\left[\sqrt{\frac{6}{5}t}\left(z+\Delta_z\right)-\frac{33}{50}t,\frac{1}{2}\chi-\frac{1}{4}t\right)$ : (i) the regulator sets access tariffs  $(K_o^*, K_n^*, \alpha_o^*, \alpha_n^*)$  on  $\left(0, \pi_i^{m_n}\left(\Delta_z\right) - I - \frac{1}{2}t\right) \times \left[0, \frac{1}{2}t\right) \times \{0\} \times \{\sqrt{6t}\}$ , (ii) the incumbent invests in the new technology, (iii) the entrant exits the market, and (iv) the incumbent sets retail tariffs  $F_i^* = F_i^{m_n}(\Delta_z)$ , and  $p_i^* = 0$ .

(IV) If I is on  $\left[\frac{1}{2}\chi - \frac{1}{4}t, \overline{I}\right)$  (i) the regulator sets access tariffs  $(\Delta_K^*, \alpha_o^*, \alpha_n^*)$  on  $[0, I] \times \{0\} \times \{0\}$ , (ii) the incumbent does not invest in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs  $F_i^* = t$ ,  $F_e^* = t$ , and  $p_i^* = 0$ ,  $p_e^* = 0$ .

Proposition 1c is similar to Proposition 1b except for part (III). If the increase in quality enabled by the new network is large, investment is socially desirable, even for high values of the investment cost, i.e., for I on  $\left[\sqrt{\frac{6}{5}t}\left(z+\Delta_z\right)-\frac{33}{50}t,\frac{1}{2}\chi-\frac{1}{4}t\right)$ . To induce investment, raising the marginal access price above marginal cost is no longer enough. The regulator must set the access tariffs such that the entrant exits the industry, which leads to a monopoly on the new network. This amounts to a *regulatory moratorium*. The regulator prefers to sacrifice lower retail prices to induce investment in the new network.

Figure 2 illustrates the four cases of Proposition 1c.<sup>16</sup>

### 4 Equilibrium of the No-Commitment Game

In this Section, we characterize the equilibria of the no-commitment game. Since the retail price game and entrant's decision of whether to stay in the market are identical to those of the commitment game, they are omitted, and we proceed directly to stage 3: the regulation of the new network.

#### 4.1 Regulation of the New Network

Next, we characterize the socially optimal access tariff to the new network, assuming that it has already been deployed, i.e., assuming that the investment cost has already been sunk.

The next Lemma presents the socially optimal access tariff to the new network.

**Lemma 5:** For the no-commitment game, the socially optimal access tariff to the new network is  $(K_n^*, \alpha_n^*)$  on  $[0, \frac{1}{2}t] \times \{0\}$ .

<sup>&</sup>lt;sup>16</sup>The corresponding Figure for Proposition 1b would have  $\widetilde{I}$  below  $\sqrt{\frac{6}{5}t}(z + \Delta_d) - \frac{33}{50}t$ . Case (II) would stop at  $\widetilde{I}$ , case (IV) would start at  $\widetilde{I}$ , and case (III) would not exist.

If  $(K_n^*, \alpha_n^*)$  is on  $\left[0, \frac{1}{2}t\right) \times \{0\}$ , the entrant asks for access to the new network. In addition, given Lemmas 1 and 3, the entrant charges the lowest possible retail price per minute, and firms charge the same retail tariff. Thus, they share the market equally, which minimizes the consumers' transportation costs.

#### 4.2 Investment Decision

Next, we characterize the incumbent's optimal investment decision. Denote by  $\Delta \Pi^{nc} (\Delta_z, K_n^*, 0, K_o, \alpha_o; I)$ , the net incremental profit of the investment for the no-commitment game, i.e.,

$$\Delta \Pi^{nc}(\Delta_z, K_n^*, 0, K_o, \alpha_o; I) := \begin{cases} \pi_i^{d_n}(\Delta_z, 0; I) - \pi_i^{d_o}(0, \alpha_o; 0) + K_n^* - K_o & \text{for } \alpha_o \text{ on } [0, \sqrt{6t}) \\ \pi_i^{d_n}(\Delta_z, 0; I) - \pi_i^{m_o}(0; 0) + K_n^* & \text{for } \alpha_o \text{ on } [\sqrt{6t}, z) . \end{cases}$$

In equilibrium, the incumbent invests in the new network, if and only if,  $\Delta \Pi^{nc} (\Delta_z, K_n^*, 0, K_o, \alpha_o; I) > 0.$ 

#### 4.3 Regulation of the Old Network

Next, we characterize the socially optimal access tariff to the old network, which is presented in the next Lemma.

**Lemma 6:** For the no-commitment game, the socially optimal access tariff to the old network is:

$$(K_o^*, \alpha_o^*) \text{ on } \begin{cases} \left[0, \frac{1}{2}t - I\right) \times \{0\} & \text{ for } I \text{ on } \left[0, \frac{1}{2}\chi\right) \\ \left[\max\left\{\frac{1}{2}t - I, 0\right\}, \frac{1}{2}t\right) \times \{0\} & \text{ for } I \text{ on } \left[\frac{1}{2}\chi, +\infty\right). \end{cases}$$

If the regulator wants to induce no-investment, he sets  $\alpha_o = 0$  and  $K_o$  such the incumbent obtains a zero incremental profit from the investment, i.e., such that  $\Delta_K \leq I$ , whatever the value of  $K_n$ . If the regulator wants to induce investment, he sets  $\alpha_o = 0$  and  $K_o$  such that the incumbent obtains a positive incremental profit from the investment, i.e., such that  $\Delta_K > I.^{17}$ 

<sup>&</sup>lt;sup>17</sup>In the no-commitment game, the regulator sets  $K_n$  after the investment, and any value  $K_n$  is an equilibrium because it is a transfer between firms, neutral in terms of welfare, and with no consequences for

Now, and contrary to the results of the commitment game, the access tariff to the old network plays a role. The reason is that the regulator now needs to set a high  $K_o$  to induce no-investment. Otherwise, the incumbent might invest when it is socially undesirable.<sup>18</sup>

#### 4.4 Equilibrium of the Whole Game

Having solved all the five stages of the no-commitment game, we can now summarize the equilibria of the whole game, which we present in the next Proposition for further reference.

**Proposition 2:** The no-commitment game has three types of equilibria:

(I) If I is on  $(0, \min\{\frac{1}{2}t, \frac{1}{2}\chi\})$  there are two types of equilibria:

(a) (i) the regulator sets an access tariff for the old network  $(K_o^*, \alpha_o^*)$  on  $[0, \frac{1}{2}t - I] \times \{0\}$ , (ii) the incumbent invests in the new technology, (iii) the regulator sets an access tariff to the new network  $(K_n^*, \alpha_n^*)$  on  $(K_o^* + I, \frac{1}{2}t) \times \{0\}$ , (iv) the entrant stays in the market, and (v) the incumbent and the entrant set retail tariffs  $F_i^* = t$ ,  $F_e^* = t$ , and  $p_i^* = 0$ ,  $p_e^* = 0$ .

(b) (i) the regulator sets an access tariff for the old network  $(K_o^*, \alpha_o^*)$  on  $[0, \frac{1}{2}t - I] \times \{0\}$ , (ii) the incumbent does not invest in the new technology, (iii) the regulator sets an access tariff to the new network  $(K_n^*, \alpha_n^*)$  on  $[0, K_o^* + I] \times \{0\}$ , (iv) the entrant stays in the market, and (v) the incumbent and the entrant set retail tariffs  $F_i^* = t$ ,  $F_e^* = t$ , and  $p_i^* = 0$ ,  $p_e^* = 0$ .

(II) If I is on  $\left[\min\left\{\frac{1}{2}t, \frac{1}{2}\chi\right\}, \overline{I}\right)$  there is one type of equilibria: (i) the regulator sets an access tariff to the old network  $(K_o^*, \alpha_o^*)$  on  $\left[\max\left\{\frac{1}{2}t - I, 0\right\}, \frac{1}{2}t\right] \times \{0\}$ , (ii) the incumbent does not invest in the new technology, (iii) the regulator sets an access tariff to the new network  $(K_n^*, \alpha_n^*)$  on  $\left[0, \frac{1}{2}t\right] \times \{0\}$ , (iv) the entrant stays in the market, and (v) the incumbent and the entrant set retail tariffs  $F_i^* = t$ ,  $F_e^* = t$ , and  $p_i^* = 0$ ,  $p_e^* = 0$ .

As in the commitment game, if the investment cost is low, i.e., if I is on  $\left(0, \min\left\{\frac{1}{2}t, \frac{1}{2}\chi\right\}\right)$ , the regulator can set the marginal access price equal to marginal cost, and use the fixed access fee to induce investment. However, at the stage where the regulator decides the access tariff for the new network, he is indifferent between setting any value of  $K_n$  on  $\left[0, \frac{1}{2}t\right)$ . If the regulator sets  $K_n$  such that the incremental profit of the investment covers the investment the ensuing stages. Since  $K_n \leq \frac{1}{2}t$ , if the regulator wants to induce no-investment, it must be that even if  $K_n = \frac{1}{2}t$ , we have  $K_n - K_o < I$ , or  $K_o > \frac{1}{2}t - I$ . In the commitment game only  $\Delta_K$  is relevant; not the value of  $K_o$ .

 $<sup>^{18}</sup>$ In Gans (2001) the payoffs before investment are given.

cost,  $\Delta_K > I$ , the incumbent invests, i.e., there is an *investment* equilibrium. If, however, the regulator sets  $K_n$  such that the incremental profit of the investment does not cover the investment cost  $\Delta_K \leq I$ , the incumbent does not invest, i.e., there is a *no-investment* equilibrium. This happens even though it is possible to induce investment with the marginal access price at marginal cost, i.e., I is on  $(0, \frac{1}{2}t)$ , and investment is socially desirable, i.e., Iis on  $(0, \frac{1}{2}\chi)$ .<sup>19</sup>

If the investment cost is high, i.e., I is on  $\left[\min\left\{\frac{1}{2}t, \frac{1}{2}\chi\right\}, \overline{I}\right)$ , there is no investment. This occurs either because investment is not socially desirable, or because it is impossible to induce investment with the marginal access price at marginal cost. Indeed, if I is high, the fixed access fee is not enough to induce investment. The incumbent only invests if the marginal access price is set above marginal cost. However, once the investment is made, it is socially optimal to set the marginal access price at marginal cost. Foreseeing that it will be expropriated from the incremental profits of the investment, the incumbent does not invest. In other words, if I is high there is a dynamic consistency problem that causes the incumbent to reduce investment.

### 5 Comparison of the Equilibria of the Two Games

In this Section, we compare the equilibria of the commitment and the no-commitment games, and discuss some policy implications.

The comparison of Propositions 1 and 2 leads to the next Corollary.

**Corollary 1:** (i) If I is on  $(0, \min\{\frac{1}{2}t, \frac{1}{2}\chi\})$ , the commitment and the no-commitment games may have the same the equilibrium, where the incumbent invests in the new network and the entrant stays in the market. (ii) If I is on  $[\frac{1}{2}\chi, \overline{I})$ , the commitment and the no-commitment games have the same equilibrium, where the incumbent does not invest.

If the investment cost is low, i.e., if I is on  $\left(0, \min\left\{\frac{1}{2}t, \frac{1}{2}\chi\right\}\right)$ , with two-part access tariffs

<sup>&</sup>lt;sup>19</sup>If one assumes that, when indifferent, the regulator sets an access price that ensures non-negative profits to the incumbent, the no-investment equilibrium disappears. If, on the other hand, one assumes that, when indifferent, the regulator favours the entrant, the investment equilibrium disappears. Since the regulator is indifferent between setting any value of  $K_n$  on  $[0, \frac{1}{2}t)$ , one can assume a random choice with expected fixed fee  $K^e = \frac{1}{4}t$ . The incumbent would invest in the new network if I is on  $(0, \min\{\frac{1}{4}t, \frac{1}{2}\chi\})$ , and would not invest if I is on  $[\min\{\frac{1}{4}t, \frac{1}{2}\chi\}, \overline{I})$ . The no-investment equilibrium disappears for low values of I. However, the range of values for I that result in investment shrinks if  $\frac{1}{4}t < \frac{1}{2}\chi$ .

it is possible, and socially desirable, to set the marginal access price equal to marginal cost and use the fixed access fee to induce investment. In these circumstances, the equilibria of the commitment and the no-commitment coincide, and two-part access tariffs solve the dynamic consistency problem. However, since the regulator is indifferent between setting any  $K_n$  on  $[0, \frac{1}{2}t)$ , he may also set a low value for the fixed access fee after investment. In this case, the equilibria of the commitment and the no-commitment do not coincide, and the dynamic consistency problem reemerges.

If the investment cost is high, i.e., if I is on  $\left[\frac{1}{2}\chi,\overline{I}\right)$ , investment is not socially desirable, even when it is possible to induce. In this case, the equilibria of the commitment and the no-commitment again coincide, trivially, and there is a duopoly on the old network.

If the investment cost takes intermediate values, i.e., if I is on  $\left[\min\left\{\frac{1}{2}t, \frac{1}{2}\chi\right\}, \frac{1}{2}\chi\right)$ , twopart access tariffs do not solve the dynamic consistency problem. The maximum admissible value for the fixed access fee, the entrant's profit, is not enough to induce investment, and the regulator has to raise the marginal access price above the marginal cost. However, after the network is deployed, it is socially optimal to set the marginal access price equal to marginal cost to promote competition on the retail market. In other words, there is a dynamic consistency problem.

To sum up, two-part access tariffs may solve the regulator's dynamic consistency problem and promote efficient investment for low values of the investment cost, and provided that the investment equilibria are reached. This contrasts with Gans (2001), where the two-part access tariffs always solve the dynamic consistency problem.

The next Corollary highlights the magnitude of the fixed access fee for the new network required to induce investment.

**Corollary 2:** If I is on  $(0, \min\{\frac{1}{2}t, \frac{1}{2}\chi\})$ , at investment equilibria, the access tariff to the new network is no smaller than the investment cost.

Having the entrant pay to the incumbent an amount no smaller than the cost of the investment goes beyond the policies of sharing the investment cost adopted by some regulators, as suggested in Gans (2001). It is questionable that the entrant would have the financial ability to make such large payments. And even if it did, it is questionable whether it would be possible, politically, to implement such a solution. Thus, even if for some parameter values two-part tariffs could solve the dynamic consistency problem of the regulation of next generation networks, it is unclear that in practice that will occur. The result of Corollary 2 takes an extreme form due to the simplicity of our model. In Hotelling's model, if the firms have the same marginal costs, i.e., if  $\alpha_v = 0$ , the firms' profits depend only on the difference in the quality of the products, and not on the absolute value of the products' quality. Thus, if both firms offer products through the new network, they have the same profit levels as when they both offer products through the old network, and consumers get all the benefits of the investment. The fixed fee must then cover the investment cost to induce the incumbent to invest.

Next, we explain how the model could be modified so that the regulator's inability to commit does not imply setting such a high fixed access fee.

First, suppose that the product of the incumbent on the new network is of higher quality than the product of the entrant.<sup>20</sup> In these circumstances, the incumbent appropriates some of the rents from the investment through a channel different from the fixed fee.<sup>21</sup>

Second, suppose that the market is not covered when the firms use only the old network. In these circumstances, the increase in the quality of the products brings new consumers to the market, increasing the firms' profits. Hence, even though the incumbent would not gain any rents with the entrant's consumers if the marginal access price is set at marginal cost, it would earn rents with its new consumers.<sup>22</sup>

These modifications complicate the model, and particularly the exposition, but do not change qualitatively our results.

### 6 Conclusion

In this article, we analyzed if two-part access tariffs can solve the dynamic consistency problem of the regulation of Next Generation Networks. With two-part access tariffs the regulator has two instruments. This might enable him to give the incumbent incentives to invest, even when he cannot commit to a regulatory policy. However, surprisingly, two-part access tariffs may only solve the dynamic consistency problem under some circumstances. Furthermore, for those circumstances, inducing investment may involve setting the fixed fee of the access tariff at level that may be politically unacceptably high. Interestingly, we identify circumstances where a regulatory moratorium emerges as socially optimal, if the regulator is able to commit to a regulatory policy.

<sup>&</sup>lt;sup>20</sup>This could happen if the incumbent had a relatively higher ability to convert the infrastructure investment into new services valued by consumers.

<sup>&</sup>lt;sup>21</sup>This is the assumption on Foros (2004) and Kotakorpi (2006).

 $<sup>^{22}</sup>$ This is similar to the result of DeBijl and Peitz (2004).

# Appendix

Lemma 1: See Biglaiser and DeGraba (2001).

**Lemma 2**: We first analyze the case where the entrant is a monopolist in the retail market using network v = o, n. Consumers purchase if and only if

$$\frac{\left(z+\Delta_{v}\right)^{2}}{2}-tx-F_{i}>0 \Leftrightarrow x<\frac{1}{t}\left[\frac{1}{2}\left(z+\Delta_{v}\right)^{2}-F_{i}\right].$$

Assuming an interior solution, the profit maximizing fixed fee and respective profits (excluding the investment cost) are

$$F_i^{m_v} (\Delta_v) = \frac{(z + \Delta_v)^2}{4}$$
$$\pi_i^{m_v} (\Delta_v) = \frac{(z + \Delta_v)^4}{16t}.$$

However, we do not have an interior solution since, given our assumption on z,

$$x^{m_v} = \frac{(z + \Delta_v)^2}{4t} > 1.$$

In this case, the optimal fixed charge and profits are:

$$F_i^{m_v}\left(\Delta_v\right) = \pi_i^{m_v}\left(\Delta_v\right) = \frac{\left(z + \Delta_v\right)^2}{2} - t.$$

**Lemma 3:** We start by finding the consumer who is indifferent between buying from the incumbent or from the entrant when both firms use network v = o, n:

$$\frac{(z + \Delta_v)^2}{2} - tx - F_i = \frac{(z + \Delta_v - \alpha_v)^2}{2} - t(1 - x) - F_e \Leftrightarrow x(F_i, F_e, \Delta_v, \alpha_v) = \frac{1}{2} - \frac{F_i - F_e}{2t} - \frac{(z - \alpha_v + \Delta_v)^2 - (z + \Delta_v)^2}{4t}$$

with  $\alpha_v < z + \Delta_v$ .

Given this indifferent consumer, and the fact that  $p_i = 0$  and  $p_e = \alpha_v$ , profit functions, excluding the investment cost, become:

$$\pi_{i} = F_{i}x\left(F_{i}, F_{e}, \Delta_{v}, \alpha_{v}\right) + \alpha_{v}\left(z + \Delta_{v} - \alpha_{v}\right)\left[1 - x\left(F_{i}, F_{e}, \Delta_{v}, \alpha_{v}\right)\right]\right)$$
  
$$\pi_{e} = F_{e}\left[1 - x\left(F_{i}, F_{e}, \Delta_{v}, \alpha_{v}\right)\right].$$

Maximizing each profit function with respect to the fixed fee, we find:

$$F_i^{d_v} \left( \Delta_v, \alpha_v \right) = t + \frac{1}{6} \alpha_v \left[ 6 \left( z + \Delta_v \right) - 5 \alpha_v \right]$$
  
$$F_e^{d_v} \left( \Delta_v, \alpha_v \right) = t - \frac{1}{6} \alpha_v^2$$

The indifferent consumer is given by

$$x^{d_v} = \frac{1}{2} + \frac{\alpha_v^2}{12t},$$

with  $\alpha_v \leq \sqrt{6t}$ .

Equilibrium profits are then:

$$\pi_i^{d_v} \left( \Delta_v, \alpha_v \right) = \frac{\left( 36t^2 + \alpha_v^4 - 60t\alpha_v^2 \right) + 72\alpha_v t \left( z + \Delta_v \right)}{72t}$$
$$\pi_e^{d_v} \left( \Delta_v, \alpha_v \right) = \frac{\left[ 6t - \alpha_v^2 \right]^2}{72t}.$$

Regarding consumers, we have to ensure that all consumers have a positive surplus, independently of the network in use.

$$\frac{\left(z+\Delta_{v}\right)^{2}}{2}-tx^{d_{v}}-F_{i}^{d_{v}} > 0 \Leftrightarrow$$

$$\left(-2\Delta_{v}\left(2\alpha_{v}-2z-\Delta_{v}\right)-4z\alpha_{v}-6t+2z^{2}+3\alpha_{v}^{2}\right) > 0.$$

This expression is minimized when  $\Delta_v = 0$  at  $(-4z\alpha_v - 6t + 2z^2 + 3\alpha_v^2) > 0$ . Given that  $z > \max\left\{\alpha_v, \frac{4}{3}\sqrt{6t}\right\}$  this is always verified.

For  $\alpha_v > \sqrt{6t}$ , the indifferent consumer is at  $x^{d_v} > 1$ , and therefore we do not have an interior solution. In this case, the optimal fixed fees and profits are:

$$\begin{aligned} \pi_i^{d_v} \left( \Delta_v, \alpha_v \right) &= F_i^{d_v} = \alpha_v \left( z + \Delta_v \right) - \frac{1}{2} \alpha_v^2 - t \\ \pi_e^{d_v} \left( \Delta_v, \alpha_v \right) &= F_e^{d_v} = 0, \end{aligned}$$

Lemma 4: See Proposition 1.

**Proposition 1:** The incumbent invests for  $\Delta \Pi^{c}(\Delta_{z}, K_{n}, \alpha_{n}, K_{o}, \alpha_{o}; I) > 0$ , with  $\Delta \Pi^{c}(\Delta_{z}, K_{n}, \alpha_{n}, K_{o}, \alpha_{o}; I) :=$ 

$$\begin{aligned} \pi_i^{d_n} \left( \Delta_z, \alpha_n; I \right) + K_n - \pi_i^{d_o} \left( 0, \alpha_o; I \right) - K_o & \text{for } (\alpha_o, \alpha_n) \text{ on } \left[ 0, \sqrt{6t} \right) \times \left[ 0, \sqrt{6t} \right) \\ \pi_i^{d_n} \left( \Delta_z, \alpha_n; I \right) + K_n - \pi_i^{m_o} \left( 0; I \right) & \text{for } (\alpha_o, \alpha_n) \text{ on } \left[ \sqrt{6t}, z \right) \times \left[ 0, \sqrt{6t} \right) \\ \pi_i^{m_n} \left( \Delta_z; I \right) - \pi_i^{d_o} \left( 0, \alpha_o; I \right) - K_o & \text{for } (\alpha_o, \alpha_n) \text{ on } \left[ 0, \sqrt{6t} \right] \times \left[ \sqrt{6t}, z + \Delta_z \right) \\ \pi_i^{m_n} \left( \Delta_z; I \right) - \pi_i^{m_o} \left( 0; I \right) & \text{for } (\alpha_o, \alpha_n) \text{ on } \left[ \sqrt{6t}, z \right) \times \left[ \sqrt{6t}, z + \Delta_z \right) \end{aligned}$$

If the regulator does not want to induce investment he should set  $\alpha_o = \alpha_n = 0$  and  $\Delta_K < I$ . If the regulator wants to induce investment, he should set  $(K_n, \alpha_n, K_o, \alpha_o)$  such that the net incremental profit of the investment is non-negative.

We start by deriving the expression for  $\hat{\alpha}_n(I)$ , the optimal access price to the new network that leads to investment.

To create conditions conducive to investment, the regulator should set  $(K_o, \alpha_o) = (0, 0)$ , so that the pre-investment profit is the lowest possible. With entry, the regulator can set, at most,  $K_n = \pi_e^{d_n} (\Delta_z, \alpha_n) - \varepsilon$ , with  $\varepsilon \to 0^+$ , for  $\alpha_n < \sqrt{6t}$ .

Under these conditions, investment occurs if and only if  $I \leq I^c(\alpha_n)$  with:

$$\begin{split} I^{c}(\alpha_{n}) &= \\ \begin{cases} \pi_{i}^{d_{n}} \left(\Delta_{z}, \alpha_{n}; 0\right) + \pi_{e}^{d_{n}} \left(\Delta_{z}, \alpha_{n}\right) - \pi_{i}^{d_{o}} \left(0, 0; 0\right) & \alpha_{n} \text{ on } \left[0, \sqrt{6t}\right) \\ \pi_{i}^{m_{n}} \left(\Delta_{z}, 0\right) - \pi_{i}^{d_{o}} \left(0, 0; 0\right) & \alpha_{n} \text{ on } \left[\sqrt{6t}, z + \Delta_{z}\right) \end{cases} = \\ \begin{cases} \frac{1}{2}t + \frac{\alpha_{n} \left(36t(z + \Delta_{n} - \alpha_{n}) + \alpha_{n}^{3}\right)}{36t} & \alpha_{n} \text{ on } \left[0, \sqrt{6t}\right) \\ \overline{I} & \alpha_{n} \text{ on } \left[\sqrt{6t}, z + \Delta_{z}\right). \end{cases} \end{split}$$

Clearly,  $I^{c}(\alpha_{n})$  is increasing in  $\alpha_{n}$  until  $\sqrt{6t}$  and then is constant.<sup>23</sup> Let

$$B_n(I) := \begin{cases} \{\alpha_n : \alpha_n \ge 0\} & I \in \left[0, \frac{1}{2}t\right) \\ \{\alpha_n : \alpha_n \ge b_n(I)\} & \text{for } I \in \left[\frac{1}{2}t, (z + \Delta_z)\sqrt{6t} - \frac{9}{2}t\right) \\ \{\sqrt{6t}\} & I \in \left[(z + \Delta_z)\sqrt{6t} - \frac{9}{2}t, \frac{(z + \Delta_z)^2 - 3t}{2}\right], \end{cases}$$

with  $b_n(I)$  implicitly defined by  $\frac{1}{2}t + \frac{b_n(36t(z+\Delta_z-b_n)+b_n^3)}{36t} = I$ , with  $\frac{\partial b_n(I)}{\partial I} = \left(z + \Delta_z + \frac{(b_n^2 - 18t)b_n}{9t}\right)^{-1} > 0$ . For a given I,  $B_n(I)$  is the set of values for  $\alpha_n$  such that the incumbent invests.

The welfare function is quasi-convex in  $\alpha_n$ . It is decreasing between  $\alpha_n = 0$  and  $\alpha_n = \frac{3}{5}\sqrt{10t}$  and then increases until  $\alpha_n = \sqrt{6t}$ . Additionally,  $W^{d_n}\left(\Delta_z, \sqrt{\frac{6t}{5}}; I\right) = W^{d_n}\left(\Delta_z, \sqrt{6t}; I\right)$ .

 $^{23}$ Note that

$$\pi_{i}^{d_{v}}\left(\Delta_{v},\alpha_{v};I\right)+\pi_{e}^{d_{v}}\left(\Delta_{v},\alpha_{v}\right)=\frac{\alpha_{v}\left[36t\left(z+\Delta_{v}-\alpha_{v}\right)+\alpha_{v}^{3}\right]}{36t}+t-I$$

with

$$\frac{\partial \left(\pi_i^{d_v} \left(\Delta_v, \alpha_v; I\right) + \pi_e^{d_v} \left(\Delta_v, \alpha_v\right)\right)}{\partial \alpha_v} = z + \Delta_v + \frac{1}{9} t^{-1} \left(\alpha_v^2 - 18t\right) \alpha_v$$
$$\frac{\partial^2 \left(\pi_i^{d_v} \left(\Delta_v, \alpha_v; I\right) + \pi_e^{d_v} \left(\Delta_v, \alpha_v\right)\right)}{\partial \alpha_v^2} = \frac{1}{3} t^{-1} \left(\alpha_v^2 - 6t\right) < 0$$

and

$$\frac{\left. \frac{\partial \left( \pi_i^{d_v} \left( \Delta_v, \alpha_v; C \right) + \pi_e^{d_v} \left( \Delta_v, \alpha_v \right) \right)}{\partial \alpha_v} \right|_{\alpha_v = \sqrt{6t}} = z + \Delta_v - \frac{4}{3}\sqrt{6t} > 0$$

Hence,  $\frac{\partial \left(\pi_i^{d_v}(\Delta_v,\alpha_v;I)+\pi_e^{d_v}(\Delta_v,\alpha_v)\right)}{\partial \alpha_v} > 0 \text{ for all } \alpha_n \text{ on } \left[0,\sqrt{6t}\right). \text{ As } \left((z+\Delta_d)\sqrt{6t}-\frac{9}{2}t\right) - \frac{(z+\Delta_d)^2-3t}{2} = -\frac{1}{2}((z+\Delta_d)-\sqrt{6t})^2 < 0 \text{ the function shifts upwards at } \sqrt{6t}.$ 

Therefore, if the regulator wants to induce investment, he should set:

$$\widehat{\alpha}_n(I) = \begin{cases} \min B_n(I) & \min B_n(I) \le \sqrt{\frac{6}{5}t} \\ \max B_n(I) & \min B_n(I) > \sqrt{\frac{6}{5}t} \end{cases}$$

Note that  $b_n(I) = \sqrt{\frac{6t}{5}}$  if and only if  $I = \sqrt{\frac{6}{5}t} (z + \Delta_z) - \frac{33}{50}t > 0$ . For higher values of I we have  $b_n(I) > \sqrt{\frac{6t}{5}}$ . Thus,

$$\widehat{\alpha}_{n}(I) = \begin{cases} 0 & I \leq \frac{1}{2}t \\ b_{n}(I) & \text{if } \frac{1}{2}t < I \leq \sqrt{\frac{6}{5}t} \left(z + \Delta_{z}\right) - \frac{33}{50}t \\ \sqrt{6t} & I > \sqrt{\frac{6}{5}t} \left(z + \Delta_{z}\right) - \frac{33}{50}t \end{cases}$$

Note that with respect to the fixed fee,  $K_n$ , we need not impose  $K_n = \pi_e^{d_n} (\Delta_z, \alpha_n) - \varepsilon$ when  $\widehat{\alpha}_n(I) = 0$ . In this case, any  $\Delta_K > I$  suffices.

We now describe under which conditions the regulator wants investment to occur.

If there is no investment, we will have  $\alpha_o = \alpha_n = 0$  and welfare will be  $W^{d_o}(0,0;I)$ . If the regulator induces investment, he will set  $\alpha_o = 0$  and  $\alpha_n = \hat{\alpha}_n(I)$  and welfare will be,  $W^{d_n}(\Delta_z, 0; I)$  if  $I \in [0, \frac{1}{2}t]$  or  $W^{d_n}(\Delta_z, b_n(I); I)$  if  $I \in (\frac{1}{2}t, \sqrt{\frac{6}{5}t}(z + \Delta_z) - \frac{33}{50}t)$  or  $W^{m_n}(\Delta_z; I)$  if  $I \in [\sqrt{\frac{6}{5}t}(z + \Delta_z) - \frac{33}{50}t, \overline{I}]$ . Comparing welfare with investment with welfare without investment in the three intervals, we conclude that the regulator prefers to induce investment if and only if:

$$I < \frac{1}{2}\chi \qquad I \in [0, \frac{1}{2}t]$$

$$I < \frac{1}{2}\chi + \frac{5(b_n(I))^2 - 36t}{144t} (b_n(I))^2 \text{ for } I \in \left(\frac{1}{2}t, \sqrt{\frac{6}{5}t} (z + \Delta_z) - \frac{33}{50}t\right)$$

$$I < \frac{1}{2}\chi - \frac{1}{4}t \qquad I \in \left[\sqrt{\frac{6}{5}t} (z + \Delta_z) - \frac{33}{50}t, \overline{I}\right],$$

As in the main text, we will now divide the proof in three cases, depending on the value taken by  $\chi$ .

Proposition 1a: Assume that  $\chi < t$ . Then, for  $\frac{1}{2}\chi < I < \frac{1}{2}t$  the regulator prefers not to induce investment although he could do so with  $\alpha_n = 0$ . This means that for larger values of I the regulator will also prefer not to induce investment.

Before proceeding to Propositions 1b and 1c, we need to introduce some notation.

Let  $\widetilde{I}$  be such that  $g(\widetilde{I}) := \widetilde{I} - \frac{1}{2}\chi - \frac{5b_n(\widetilde{I})^2 - 36t}{144t} \left(b_n(\widetilde{I})\right)^2 = 0$ . As  $\frac{\partial g(I)}{\partial I} > 0$ , we have that  $I < \frac{1}{2}\chi + \frac{5(b_n(I))^2 - 36t}{144t} (b_n(I))^2$  is equivalent to  $I < \widetilde{I}$ . This means that the regulator prefers to induce investment when in order to do so he has to set  $\widehat{\alpha}_n(I) = b_n(I)$  if and only if  $I < \widetilde{I}$ .<sup>24</sup>

It can be showed that if  $\chi > t$  then  $\widetilde{I} > \frac{1}{2}t$  because  $g\left(\frac{1}{2}t\right) := \frac{1}{2}(t-\chi) < 0$  and that

<sup>&</sup>lt;sup>24</sup>Note that  $\frac{\partial g(I)}{\partial I} = \frac{3}{4} \frac{(12t(z+\Delta_d)-18tb_n+b_n^3)}{(9t(z+\Delta_d)-18tb_n+b_n^3)} > 0.$ 

 $\chi < \chi^{c} := \left(\frac{2\sqrt{15}}{25}\sqrt{50\frac{z^{2}}{t} + 19} + \frac{79}{50}\right)t \text{ if and only if } \widetilde{I} < \sqrt{\frac{6}{5}t} \left(z + \Delta_{z}\right) - \frac{33}{50}t.^{25}$ Proposition 1b. Assume that  $t < \chi < f(t, z)$ .

(I) If  $I < \frac{1}{2}t$  the regulator can induce investment without distorting the access prices. As  $\frac{1}{2}t < \frac{1}{2}\chi$  he prefers to do so and sets  $\alpha_n = 0$ , and  $\Delta_K \in (I, \frac{1}{2}t)$ .

(II) If  $\frac{1}{2}t < I < \tilde{I}$ , the regulator will distort  $\alpha_n$  to induce investment, since the investment restriction becomes binding. Thus, the regulator sets  $\Delta_K$  to the maximum, in this case,  $\pi_e^{d_n}(\Delta_z, \alpha_n) - \varepsilon$ .

(III) If  $\tilde{I} < I < \sqrt{\frac{6}{5}t} (z + \Delta_z) - \frac{33}{50}t$ , the regulator prefers that there is no investment and should set  $\alpha_o = \alpha_n = 0$  and  $\Delta_K < I$ .

If  $\sqrt{\frac{6}{5}t}(z + \Delta_z) - \frac{33}{50}t < I < \overline{I}$ , the regulator also prefers that there is no investment and should set  $\alpha_o = \alpha_n = 0$  and  $\Delta_K < I$ . This happens because as  $\widetilde{I} < \sqrt{\frac{6}{5}t}(z + \Delta_z) - \frac{33}{50}t$ , then, for  $I = \sqrt{\frac{6}{5}t}(z + \Delta_z) - \frac{33}{50}t$  we have  $W^{d_n}(\Delta_z, b_n(I); I) = W^{m_n}(\Delta_z; I)$  and  $W^{d_n}(\Delta_z, b_n(I); I) < W^{d_o}(0, 0; I)$ . This implies that  $W^{m_n}(\Delta_z; I) < W^{d_o}(0, 0; I)$  which means that  $\sqrt{\frac{6}{5}t}(z + \Delta_z) - \frac{33}{50}t > \frac{1}{2}x - \frac{1}{4}t$ . Thus, it is impossible to have  $I < \frac{1}{2}\chi - \frac{1}{4}t$ .

Proposition 1c. Assume that  $\chi > \chi^c$ .

(I) If  $I < \frac{1}{2}t$  the regulator can induce investment without distorting the access prices. Thus, he sets  $\alpha_n = 0$ , and  $\Delta_K \in (I, \frac{1}{2}t)$ .

(II) If  $\frac{1}{2}t < I < \sqrt{\frac{6}{5}t} (z + \Delta_z) - \frac{33}{50}t$ , the regulator will distort  $\alpha_n$  to induce investment, since the investment restriction becomes binding. Thus, the regulator sets  $\Delta_K$  to the maximum, in this case,  $\pi_e^{d_n} (\Delta_z, \alpha_n) - \varepsilon$ .

(III) If  $\sqrt{\frac{6}{5}t}(z + \Delta_z) - \frac{33}{50}t < I < \frac{1}{2}\chi - \frac{1}{4}t$  the regulator sets  $\alpha_n = \sqrt{6t}$  and there is investment and no entry. Contrary to case 1b, we have that  $\widetilde{I} > \sqrt{\frac{6}{5}t}(z + \Delta_z) - \frac{33}{50}t$ . Then  $I = \sqrt{\frac{6}{5}t}(z + \Delta_z) - \frac{33}{50}t$ ,  $W^{m_n}(\Delta_z; I) > W^{d_o}(0, 0; I)$ , which means that  $\sqrt{\frac{6}{5}t}(z + \Delta_z) - \frac{33}{50}t < \frac{1}{2}\chi - \frac{1}{4}t$ . The regulator must have  $\pi_i^{m_n}(\Delta_z) - I > \frac{1}{2}t + K_o$ .

(IV) If  $\frac{1}{2}\chi - \frac{1}{4}t < I < \overline{I}$  the regulator prefers that there is no investment and should set  $\alpha_o = \alpha_n = 0$  and  $\Delta_K < I$ .

**Lemma 5:** According to our analysis of the welfare function  $\alpha_n = 0$  maximizes welfare. Then the regulator just needs to assure that the entrant asks for access: That is why  $K_n < \frac{1}{2}t$ .

**Lemma 6:** As in Lemma 5 the regulator sets  $\alpha_o = 0$ , and is indifferent between setting any  $\overline{^{25}\text{As }b_n\left(\sqrt{\frac{6}{5}t}\left(z+\Delta_z\right)-\frac{33}{50}t\right)} = \sqrt{\frac{6t}{5}}$  we have that  $g\left(\sqrt{\frac{6}{5}t}\left(z+\Delta_z\right)-\frac{33}{50}t\right) > 0 \Leftrightarrow \chi < \chi^c$ . Thus,  $\chi < \chi^c$  is equivalent to  $\widetilde{I} < \sqrt{\frac{6}{5}t}\left(z+\Delta_z\right)-\frac{33}{50}t$ .

 $K_o$  on  $\left[0, \frac{1}{2}t\right]$ . If he does not want to induce investment he sets  $K_o$  such that it will never compensate for the incumbent to invest for any  $K_n$ , i.e.,  $K_n^* - K_o < I$ . Since  $K_n^* \in \left[0, \frac{1}{2}t\right)$ , he sets  $K_o \in \left[\max\left\{\frac{1}{2}t - I, 0\right\}, \frac{1}{2}t\right)$ . If he wants to induce investment, he should set  $K_o$  such that  $K_n^* - K_o > I$  for some  $K_n^* \in \left[0, \frac{1}{2}t\right)$ , which implies setting  $K_o \in \left[0, \frac{1}{2}t - I\right)$ .

**Proposition 2:** Follows from Lemmas 1, 2, 3, 5 and 6.

Corollary 1: Follows from Propositions 1 and 2.

Corollary 2: Follows from Proposition 2.

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## Figures

Figure 1



Figure 1: Welfare as a function of the access price.

Figure 2



Figure 2: The thick line represents  $\widehat{\alpha}_n(I)$ , the shaded area corresponds to the values of I such that  $I \leq I^c(\alpha_n)$ , i.e., such that investment occurs and the intervals (I) to (IV) correspond to cases (I) to (IV) in Proposition 2c.